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ABSTRACT

This study guide, designed for use at Oklahoma State University, contains lists of activities for students to perform based on the "mastery of learning" concept. The activities include readings, problems, self evaluations, and assessment tasks. The units included are: Functions, Exponential and Logarithmic Functions, Trigonometric Functions, Polynomial Functions, Trigonometric Equations and Identities, Numerical Trigonometry, Combinatorics, Probability, Linear Algebra, Complex Numbers, and Sequences. (MK)

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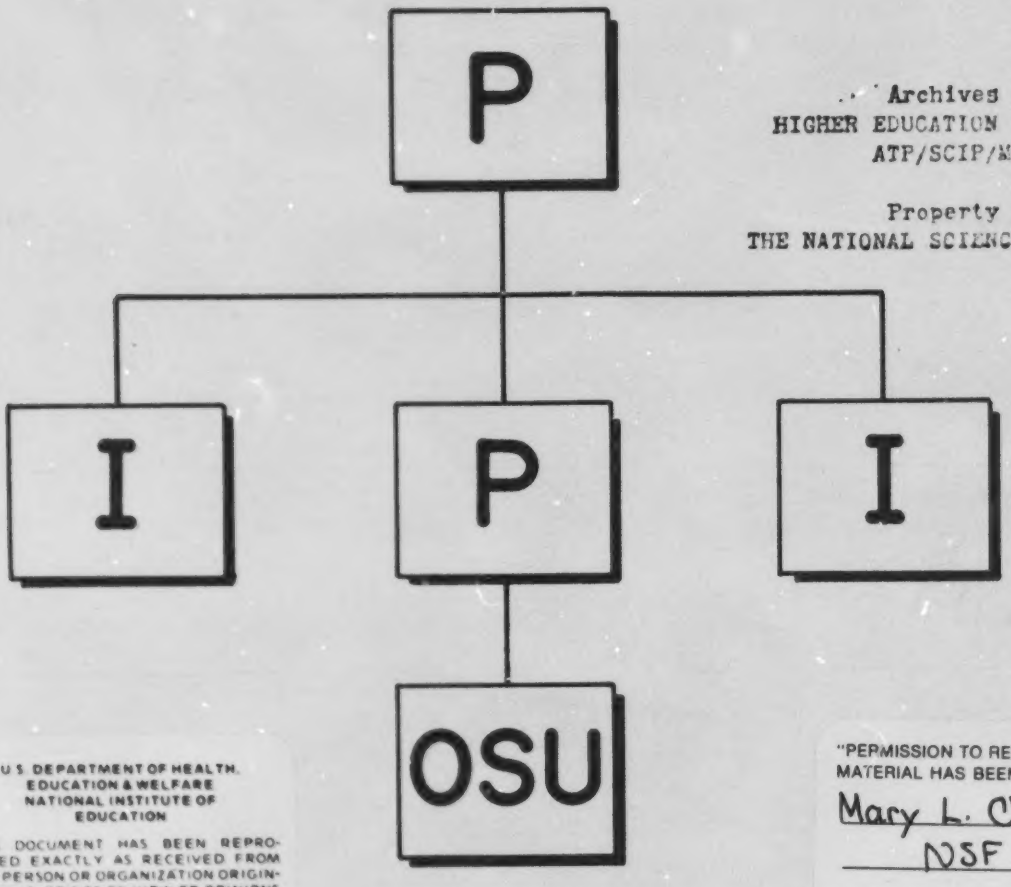

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FOREWARD

To the Student:

This manual or study guide is a list of activi ies for you to perform based on the "mastery of learning" concept. Conventional presentation of a course often permits students to continue in the mathematics sequence even when performance on a given unit or topic is not satisfactory. Grading practices may allow weakness in a unit to be overshadowed by strength in other units comprising a course. Such low level performance on a given topic often makes a future course, where an indepth study of the topic occurs, the students' last endeavor in mathematics.

The list of activities and periodic check points force sufficient competence before the next unit can be undertaken. The activities are arranged in a hierarch, of interdependence so that no missing blocks occur in your foundation of mathematics. If you will practice self evaluation using the hierarchy as a guide, not exercise your remarkable ability to cut corners and get around the hierarchy then you can make the guarantee to yourself that you have the desired foundation.

The list of activities includes readings, problems, self evaluations, and assessment tasks among others. Two activities need to be emphasized. First, reading a mathematics topic is not the same as reading a novel. Speed reading is not offered by the Mathematics Department. Several readings of a topic are often required, with pencil and paper handy. Second, problems imply practice. The problems suggested are for your practice. Saturday's game is won or lost according to the practices prior to the game.

The game is a participant's game, not a spectator's game. The game is much more enjoyable when the rules are learned. Concentrate on the rules and develop your skills, and you will be an enthusiastic participant. The whistle need not blow followed by the call of your number for violation of a rule committed in ignorance.

A BRIEF DESCRIPTION OF TERMS USED

Module. The content of this course has been organized into eleven parts called modules.

Objective. Certain tasks or skills are required to attain a sense of competency in each module. Each of these tasks or skills is spelled out for you as an objective.

Hierarchy. The modules within the course and the objectives within each module possess a particular ordering called the hierarchy. The hierarchy is illustrated in Appendix A. If you look at this diagram, you will notice that there are often optional paths that may be chosen.

Activities. There are a number of ways to attain the competency required in an objective. These are listed in the Study Guide as as activities; they include the use of your text, other reading sources, solved problems, audio tapes, lectures, individual assistance, and informal group sessions.

Reference Sources. Appendix B contains information as to the availability of the references suggested in the activities of each objective.

Lectures. A series of lectures has been developed, and they will be presented at the rate of about one per week. You will be given a schedule telling when and where the lectures will be given. For the most part a lecture will have as its topic a particular module. These lectures should be very helpful, but they are entirely optional and will have no direct bearing on your grade.

Individual Assistance. As in acquiring any new skill, you may expect some difficulties. Your instructors are available to you on an individual basis to help you overcome these difficulties.

Informal Group Sessions. There is no reason why you should always work alone. In fact, there are many benefits in working with others. You will have available to you a list of the other members of the class and their phone numbers.

Self Evaluation. After performing a sequence of activities for an objective, you will need to know whether you are ready to proceed to the next objective. The self evaluation serves this purpose. It is a self-test that can show you if you have acquired the competency required for an objective.

Assessment Task. After you feel confident that you have accomplished what was required by each objective in a module, you need to request an assessment task. This is a test that will be used by an instructor in determining whether the objectives of the module have been met satisfactorily. If they have, then you may begin another module. If they have not, you will be instructed as to how the deficiency may be overcome. You may repeat the portion of a similar assessment task that measures the area of deficiency when you again feel ready. These assessment tasks will closely resemble the self evaluations at the end of each objective.

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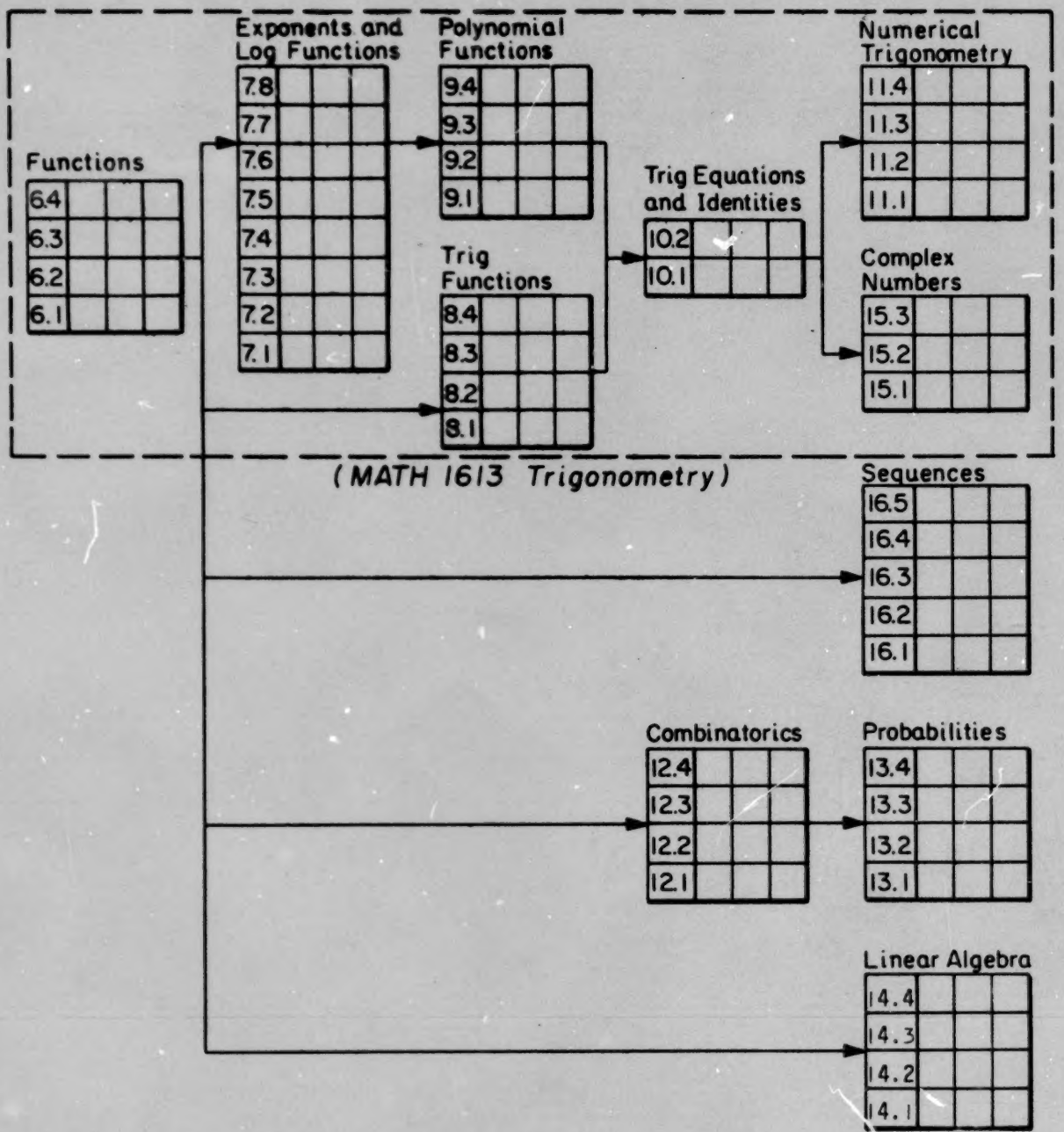
MATH 1715 (Algebra and Trigonometry)



MODULE 6

FUNCTIONS

PREREQUISITE

Competency in modules 1-5 or equivalent of one and one-half years of high school algebra and in communication units 3, 4, 10 and 11.

RATIONALE

A most important concept that appears in nearly every branch of mathematics is the concept of a function. Should you be one to pursue mathematics beyond this course, the above statement is sufficient to impress on you the importance of this module. If your goal in mathematics is of shorter range then note the titles of modules that follow: Exponential and Logarithmetic Functions, Trigonometric Functions and Polynomial Functions. Even those modules without the name "function" in the title require some use of the concept and symbolism of functions.

On your trek to the terminal objective you will gather some terminology and abbreviation for simplicity and communication, visit relations for convenience in defining functions, graph functions and relations, perform operations on functions and find inverses of functions. The thought may occur to you to consider the structural concepts of commutative, associative, distributive, etc. in terms of the set of functions. The structure is not the care of this module but will be of consideration in future courses.

MODULE ACTIVITIES

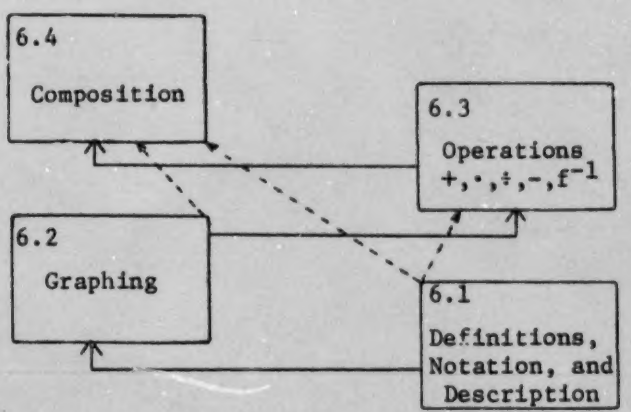
Lecture 6

The lecture over this module will have the following outline:

1. Definitions - function, relation, domain, range
2. Graphing functions and relations - distinguishing functions by graph, by rule
3. Operations - sum, product, difference, quotient, inverse and composition, domain and range.

Procedural Options

This module is an important one to you not only because it is a prerequisite for later work, but because you will develop a style and work habits that will affect your work throughout the course. Below is a graphic representation of the content pertaining to the measurable objectives mentioned.



After 6.1 you could go to any of the other objectives, but you are advised to follow the sequence because of the arrangement of the materials. You have these options:

Option 1. Select a device on the chart that follows from which you can acquire the skills and concepts for each objective, beginning with 6.1. After you have completed an activity package for each objective, take

the self evaluation. If your score is 100% and you are confident, proceed to the next objective and continue until 6.4 is completed. You are now ready for the assessment task which measures all objectives 6.1 - 6.4.

Option 2. Study the first objective carefully. If you feel confident, take the self evaluation for that objective. If you still feel confident and scored 100%, go to the next numbered objective and repeat this sequence until you lose confidence or score less than 100%. Begin with the activity package in the objective in which the "bust" occurred.

Option 3. There exists at least one other option which you may exercise to accomplish the objectives. Whatever it is, do it. Don't abuse the self evaluations. When a bust occurs, don't peek ahead and destroy the evaluation atmosphere you will have when you ask for the module assessment.

Average Time - Four days.

☐ OBJECTIVE 6.1

Can identify functions, relations which are not functions, the range and domain of relations (functions) when the relation (function) is described verbally, as a rule, or as a set of ordered pairs.

Activities 6.1 (We suggest 1 and 4B)

☐ 1. Your Text:

Robison, Modern Algebra And Trigonometry, pp. 55-62, Exercise Set 3.1, Problems 1-12, 17-19.

☐ 2. Solved Problems:

Schaum's Outline Series: Theory and Problems of College Algebra, pages 75-86, 77-78, Problems 1, 2, 4; page 85, Problems 23-26. (The outline does not discuss range, domain nor does it define a function in terms of a relation which is the approach we are taking.)

☐ 3. Other Reading Sources:

Vance: Modern College Algebra (The adopted text at OSU for College Algebra 1513) pages 91-96 and problems.

Knight and Hoff: Introduction to the Elementary Function, Chapter 2, pp. 8-11. (A good treatment though the idea "relation" is not presented.)

Most any topic, book or chapter of a book dealing with "elementary functions". Cue in on that topic heading or "relation", "function", "domain", "range".

☐ 4. Audio Tape:

A. Tape 6.1. Definition, description of functions and relations, notation.

B. Tape 32 and accompanying text, Elementary Algebra, Moon and Davis, Merrill, Columbus. (A commercial text and tape available in the library.)

☐ 5. Lecture:

Note the lecture schedule and the outline of the lecture. The lecture, of course, will cover the entire module, not just 6.1.

☐ 6. Individual Assistance:

Your instructors are available daily for individual help in the IPI Center.

☐ 7. Informal Group Session:

The project room is a good place to study with other students in this program. Get acquainted with your fellow students and make full use of this valuable source of assistance.

☐ Self Evaluation 6.1

Part I Competency: 11 correct of 13.

Direction: Classify with the letter F, R or N the following expressions according to the definition of function, relation

F if a function
R if a relation but not a function
N if neither a function nor a relation

_____ 1. $A = \{(0,1),(3,5),(5,1),(1,5)\}$

_____ 2. $B = \{(0,1),(1,1),(2,1),(3,1)\}$

_____ 3. $C = \{(1,0),(1,1),(1,2),(1,3)\}$

_____ 4. $D = \{(\text{Missouri},22),(\text{Texas},5),(c,\text{Brown}),(\text{sun},\Delta)\}$

_____ 5. $e = \{(x,f(x)) \in R \times R \mid f(x) = 3x + 2\}$

_____ 6. $F = \{3,c,\text{tree},\text{car}\}$

_____ 7. $g = \{(x,y) \mid y = 2x + 3, \quad x = -1, 2, 3\}$

_____ 8. $h = \{(x,y) \mid y = \sqrt{9 - x^2}, \quad y \in R\}$

_____ 9. $j = \{(x,y) \mid y = \frac{1}{1 - x}\}$

_____ 10. k is the rule "is the brother of" and D(k) is the set of all men.

_____ 11. $m = \{(x,y) \in A \times A \text{ where } A = \{a,2,\text{comet}\}\}$

_____ 12. A number n is the square of a whole number.

_____ 13. The sum of two whole numbers is at most 8.

Part II Competency: 7 subproblems correct of 8

14. Write the domain of the expressions in Part I

a) A (Prob. 1)
b) e (Prob. 5)
c) g (Prob. 7)
d) j (Prob. 9)

15. Write the range of

a) B (Prob. 2)
b) g (Prob. 7)
c) h (Prob. 8)
d) $r(x) = 1/\sqrt{1 - 2x}$

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 6.2

Construct graphs of relations, functions when described verbally, by a function rule or as a set of ordered pairs and can distinguish relations and functions from the graphs.

Activities 6.2 (We suggest 1 and 4)

☐ 1. Your Text:

Study pages 55-62, Problems 13-16, 20-28.

☐ 2. Other Reading Sources:

Vance: Study pages 98-101, Problems of page 100,101.

Knight and Hoff: Pages 11-15.

Davis and Moon: Elementary Algebra, Unit 32 with Audio Tape.

Other text or topic. Look for "graphs of functions".

☐ 3. Solved Problems:

Schaum's Outline: College Algebra, Chapter 10. Functions and Graphs.

☐ 4. Cassette Tapes:

Tape 6.2. Distinguishing Functions and Relations and Graphing.

☐ 5. Lecture.

☐ 6. Individual Assistance in the IPI Center.

☐ 7. Informal Group Sessions:

There is room in the IPI Center for you to work with other students.

☐ **Self Evaluation 6.2**

Part I Competency: 100%

Direction: Sketch the graph of the following relations.

1. $f(x) = \begin{cases} x^2, & x > 1 \\ -x + 2, & 0 < x < 1 \\ 1, & x \leq 0 \end{cases}$

2. $g(x) = \{(x, g(x)) \in A \times A \text{ where } A = \{-1, 1, 2\}\}$

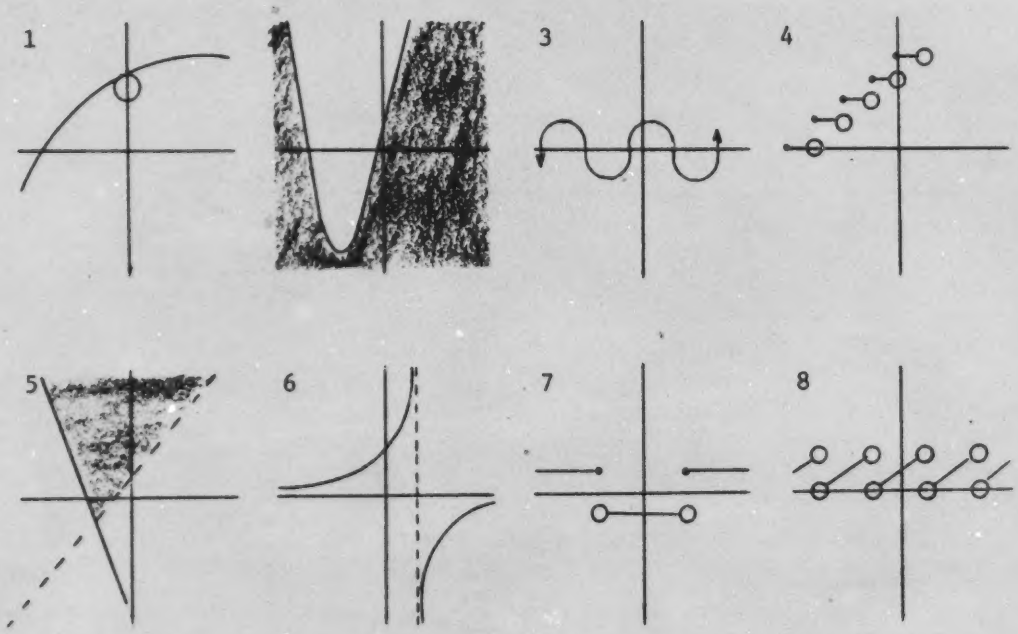
3. $h = \{(0,2), (-1,3), (2,3), (1,1)\}$

4. $y \geq 2x - 1$

5. $y = |x| - 1$ definition $|a| = \begin{cases} a & \text{when } a \geq 0 \\ -a & \text{when } a < 0 \end{cases}$

Part II Competency: 100%

Direction: Identify the following relations which are not functions.



Evaluate your progress by checking your answers in the Self Evaluation Appendix.

☐ OBJECTIVE 6.3

Name the inverse of a function, the sum, difference, product and quotient of two functions.

Activities 6.3 (We suggest 2A or 4)

☐ 1. Your Text:

The reading material and problems in the text deal with inverses and not with the arithmetic operations. The text has a good treatment on inverses. Pages 63-66 and problems 1-7 of Exercise Set 3.2.

☐ 2. Other Reading Sources:

Vance: A good complete treatment of this topic. Pages 96-98 and the problems within these pages except where you are asked to find fog or gof which is included in the next objective. See Ch. 9 Sec. 1 for inverses of polynomial functions.

Knight and Hoff: A complete treatment of the material; pages 15-26.

☐ 3. Solved Problems:

Schaum's Outline Series, Chapter 10.

☐ 4. Tapes:

A. 6.3A. Sums, Differences, Products and Quotients of Functions.

B. 6.3B. Inverses of Functions and Relations.

☐ 5. Individual Assistance:

Your instructors are available daily to assist you.

☐ 6. Informal Group Sessions.

☐ 7. Lecture.

☐ Self Evaluation 6.3

Competency: 9 correct of the 10 problems.

Directions for problems 1-7

$f(x) = 2x + 3 \quad -3 < x \leq 2$

$g(x) = 3x - 5 \quad -2 < x \leq -3$

$h(x) = 2 - x^2$

$i(x) = 3/x$

$k(x) = 2 + 3x$

Express as a single relation problems 1-5.

1. $f(x) + g(x)$ and give domain.
2. $k(x) - h(x)$ and give domain.
3. $k(x)/i(x)$ and give domain.
4. $(h(x))(k(x))$ and give domain.
5. $(k^{-1}(x))(i(x)) + h(x)$
6. Find $f^{-1}(x)$. Is the inverse a function?
7. Find the inverse of $h(x)$.
8. If $T = \{(1,3),(3,4),(4,2)\}$, graph T^{-1}.
9. Graph $k^{-1}(x)$.
10. Graph $(f + g)(x)$; i.e. $f(x) + g(x)$.

Check your answers in the Self Evaluation Appendix. If you scored at least 9 (no part credit) continue to Objective 6.4. Otherwise, review the material of the type of problems you missed.

☐ OBJECTIVE 6.4

Given functions $f(x)$ and $g(x)$, can find a third function $h(x)$ where $g(x) = f(h(x))$.

Activities 6.4 (We suggest 2A or 3)

☐ 1. Your Text:

This topic is not treated in your text. Note on notation: $f(g(x))$ may be written fog and read as "apply f after g."

☐ 2. Other Reading Sources:

Vance: pages 96-98 and problems 1-12. A good treatment.

Knight and Hoff: pages 26-29.

☐ 3. Cassette Tapes:

Tape 6.4. Composition of Functions

☐ 4. Lecture.

☐ 5. Individual Assistance in the IPI Center.

☐ 6. Informal Group Sessions.

☐ Self Evaluation 6.4

Competency: 5 correct

If $f(x) = 4 - x$

$g(x) = 2x + 3$

$h(x) = x^2 - 2x$

1. Evaluate $f(g(3))$.
2. Express $f \circ g$ as a single function.
3. Express $g(h(x))$ as a single function.
4. If $f(r(x)) = g(x)$, find $r(x)$.
5. If $D_f = 1 < x < 4$, find the range of $(h \circ f)(x)$.

Evaluate your progress by checking the solution in the Self Evaluation Appendix.

ASSESSMENT TASK 6

If you have completed the activity packages and met the self evaluation hurdles, have a sense of confidence in the content of this module, you are ready to take the assessment task over the entire module. It is put together somewhat like the four self evaluations lumped together as one evaluation. You will need pencil, paper and graph paper. Ask for Module 6 Assessment. We believe you will do well. The assessment takes approximately one hour. If you have time and the instructor is free, your assessment may be checked soon after you take it.

WHAT NEXT

We hope you demonstrated competence in each group of the module assessment. The instructional activities were designed to provide sufficient practice for mastery of this foundation module.

If you performed well, begin Module 7 with a smile and the confidence that you are prepared. You are prepared for the future activities of the hierarchy demanding proficiency in functions for success. Note, however, Module 7 continues the study of elementary functions.

Perhaps you just squeaked through. In that event, smile and begin Module 7. You may have to brush up on the material at a later date, but it takes only a short time to refresh yourself on material you have once learned.

You shouldn't be discouraged if you bombed the assessment. Your instructor will give you the green light for Module 7 as soon as you strengthen the weakness indicated by the assessment. You will need only to recycle through the activities and the assessment where the bust occurred (we will ask you to take the complete module assessment on the third recycle if that should happen). Try again. We believe you will perform well if you do the suggested activities. Select a set of activities different from your earlier choice. Your instructor will help you choose a path if you will ask.

A word on procedure. Suppose you did not demonstrate proficiency. On your second trial you will be asked to clear up the objectives you missed. If on your second trial you again do not make it, you will be asked to redo the complete assessment. This procedure will be followed throughout the program. Odd trials do all; even trials, clear up the portion missed on the previous trial. In all cases you will see a different assessment form.

Another word on procedure. Do not ask to take an assessment on the same day you review a "busted" assessment.

MODULE 7

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

PREREQUISITE

Competency in modules 1-6.

RATIONALE

The functions that have been considered in the preceding modules were mainly algebraic functions, that is, functions which can be defined by equations of the form $f(x) = p$, where p is an algebraic expression in x. Functions which are not algebraic are called transcendental. In this module we shall define two families of transcendental functions, exponential functions and logarithmic functions; and we shall investigate some of the elementary properties of these functions.

Exponential functions are extremely significant in higher mathematics. They are also of considerable importance in science and technology. Any quantity which increases by a fixed percent of itself at regular intervals is said to increase exponentially. This phenomenon is referred to as the Compound Interest Law when experienced in economics or the Law of Organic Growth when experienced by the physical scientist.

The topic of logarithms has changed considerably in the past several years. A century ago the principal reason for studying logarithms was its use as a tool in carrying out long computations quickly and easily. At present, such computations may be done with slide rules when a small degree of accuracy is needed; otherwise, high speed computers may be utilized. The significance of the logarithmic function is its role as a function with important properties and its relation to the exponential function.

Due to the length of this module and the natural division that exists, Module 7 will be divided into two submodules: 7.A - Exponential Functions and 7.B - Logarithmic Functions.

SUBMODULE 7.A - Exponential Functions

SUBMODULE ACTIVITIES

Lecture

This lecture will begin with a discussion of positive integral exponents and their properties. The theory will then be generalized to include all integral exponents, then all rational exponents, and finally the exponential function. The utilization of exponents to solve radical equations and the graphing of exponential functions will be demonstrated.

Note: Check the lecture schedule to see whether lecture 7.A will be presented at a beneficial time for you. If it will not, don't wait, you should be able to successfully complete the objectives of this submodule without relying on the lecture.

If you elect to attend the lecture, it is essential that you complete the reading assignments in your text for this submodule prior to the lecture.

Procedural Options

You may choose to accomplish all the objectives of this submodule by relying on the sequence presented in the text or following the sequence, objective by objective, as presented below.

Average Time

This submodule should take the average student three days to complete - provided there exists an average student.

☐ OBJECTIVE 7.1

Evaluate and simplify exponential expressions by applying the definition and basic laws of integral exponents.

Activities 7.1 (We suggest 1 and 4)

☐ 1. Your Text:

Chapter 6, pages 129-132. Pay close attention to the examples given on these pages. Do the odd problems on pages 131 and 132.

☐ 2. Other Reading Sources:

Vance: Modern College Algebra, Chapter 8, pages 59-60.

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 142-146.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, pages 44-45, problems 1, 2, and 5.

☐ 4. Audio Tape:

Tape 7.1. "Working with Exponents". This tape will begin with the basic properties of exponential functions. Problems in evaluating and simplifying exponential expressions will be worked in detail.

☐ 5. Individual Assistance:

Your instructors are available to you daily for individual help.

☐ 6. Informal Group Sessions.

☐ 7. Lecture 7.A.

☐ Self Evaluation 7.1

1. Perform the indicated operation, using the laws of exponents. Simplify by removing all negative exponents, and reduce all complex fractions to simple fractions.

   a) $(2^2)^3 \div 2$

   b) $b^{5n} \cdot b^{-2n}$

   c) $\pi^{-2} \div \pi^3$

2. Simplify the following:

   a) $(x^{-2})^{-3}$

   b) $\dfrac{(x^3y^{-2})^2(xy^{-1})^3}{x^{-2}y}$

   c) $\dfrac{a^{-2} - b^{-2}}{a^{-1} - b^{-1}}$

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 7.2

Evaluate and simplify exponential and radical expressions by applying the definition and basic laws of rational exponents.

Activities 7.2 (We suggest 1 and 4)

☐ 1. Your Text:

Chapter 6, pages 132-137. Work the odd numbered problems on pages 136-137. Pay close attention to the examples.

☐ 2. Other Reading Sources:

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 150-154.

Vance: Modern College Algebra, Chapter 3, pages 60-67.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, pages 44-47, problems 3, 4, 6, 7, 8, 9, 10.

☐ 4. Audio Tape:

Tape 7.1. "Working With Exponents". For description, see Activities 7.1.

☐ 5. Individual Assistance.

☐ 6. Informal Group Sessions.

☐ 7. Lecture 7.A.

☐ Self Evaluation 7.2

1. Evaluate the following:

a) $(-8)^{-\frac{2}{3}}$

b) $\sqrt[5]{\left(-\frac{1}{32}\right)^2}$

c) $4x^{-\frac{2}{3}} + 3x^{\frac{1}{3}} + 2x^0$, when $x = 8$.

2. Simplify the following:

a) $\sqrt[3]{\frac{8x^{-3}}{27y^6}}$

b) $\left(\frac{-1}{\pi}\right)^{-\frac{1}{3}}$

c) $\frac{1}{\sqrt{a} - \sqrt{b}}$

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 7.3

Solve radical equations.

Activities 7.3 (We suggest 1 and 4)

☐ 1. Your Text:

Chapter 6, pages 137-139.

☐ 2. Other Reading Sources:

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 3, pages 51-53.

Vance: Modern College Algebra, Chapter 6, pages 128-129.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, pages 118-119, problems 14-18.

☐ 4. Audio Tape

Tape 7.3. "Solving Radical Equations." Two procedures for solving radical equations will be described and demonstrated.

☐ 5. Individual Assistance.

☐ 6. Informal Group Sessions.

☐ 7. Lecture 7.A.

☐ Self Evaluation 7.3

Solve the following radical equations and check your solutions.

1. $\sqrt{4x^2 - 1} = 2x + 1$
2. $\sqrt{x^2 - 5} = x$
3. $\sqrt{x + 2} = 1 - \sqrt{x + 3}$
4. $\sqrt{x - 2} = \sqrt[4]{x}$
5. $\sqrt[3]{x^2 - x} = \sqrt[3]{2x - 2}$

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 7.4

Graph exponential functions, $f(x) = b^x$, $b > 0$, and estimate the value of the function for rational values of x by using the graph.

Activities 7.4 (We suggest 1 and 3)

☐ 1. Your Text:

Chapter 6, pages 139-143. Problems 1,2,3,6,9 on pages 142-143.

☐ 2. Other Reading Sources:

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 155-159.

Vance: Modern College Algebra, Chapter 12, pages 228-229.

☐ 3. Audio Tapes:

Tape 7.4. "Graphing Exponential Functions."

The graphs of the exponential functions $f(x) = 3^x$ and $f(x) = \left(\frac{1}{3}\right)^x$ will be constructed. The relationship between these functions will be discussed and illustrated.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 7.A.

☐ Self Evaluation 7.4

1. a) Graph the function $f(x) = 4^x$.

b) Estimate the value of $(4)^{1/3}$ using your graph.

2. a) Graph the function $f(x) = \left(\frac{1}{2}\right)^x$

b) Estimate the value of $\left(\frac{1}{2}\right)^{1/2}$ using your graph.

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

ASSESSMENT TASK 7.A

As soon as you feel that you are capable of responding correctly to questions in all parts of this submodule, make an appointment to complete an assessment task over this material.

WHAT NEXT

Your background in exponents may have been sufficient for you to skip some of the activities and still demonstrate competence in this submodule. That's great! It's the mastery of the material that is of importance, not how, when or the time interval required to attain mastery. You are now ready to begin Submodule 7B.

Suppose you did not demonstrate mastery when you took the assessment. There is no need for you to be discouraged. Try again, your instructor will assist you in mapping a learning path to enable you to attain and demonstrate mastery of this material.

SUBMODULE 7.B - Logarithmic Functions

SUBMODULE ACTIVITIES

Lecture 7.B (See Lecture Schedule)

Lecture 7.B will begin with a definition of the logarithmic function in terms of the exponential function. Graphing of logarithmic functions will be demonstrated. Basic properties of logarithms will be derived and used to simplify and evaluate logarithmic expressions and to solve logarithmic and exponential equations. The use of logarithms and the table of common logarithms in arithmetic computation will be discussed.

Procedural Options

You may choose to accomplish all the objectives of this submodule by relying on the sequence presented in the text or following the sequence, objective by objective, as presented below.

Average Time

This submodule should take about one week to complete.

☐ OBJECTIVE 7.5

Given a logarithmic statement, write the equivalent exponential statement with the appropriate restrictions, and vice versa.

Activities 7.5 (We suggest 1 and 3)

☐ 1. Your Text:
Chapter 7, pages 144-145. Problems 1,2,3 and 4 on page 148.

☐ 2. Other Reading Sources:

Eulenberg & Sunko: Introductory Algebra, Chapter 9, pages 222-225.

Vance: Modern College Algebra, Chapter 12, pages 236-238.

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 159-161.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 22, page 212, problems 1, 2, 3, and 4.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 7.B.

☐ Self Evaluation 7.5

1. Express each of the following exponential statements in logarithmic form.

a) $a^b = c, \quad a > 0;$

b) $2^5 = 32;$

c) $3^{-3} = \frac{1}{27}.$

2. Express each of the following logarithmic statements in exponential form.

a) $\log_\pi a = b;$

b) $\log_{\frac{1}{4}} \frac{1}{16} = 2;$

c) $\log_4 2 = \frac{1}{2}.$

3. Solve each of the following for x.

   a) $\log_{125} 25 = x$;

   b) $\log_x 27 = 3$;

   c) $\log_2 x = -5$.

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 7.6

Graph the logarithmic function, $f(x) = \log_b x$, $b > 0$, $b \neq 1$.

Activities 7.6 (We suggest 1 and 3)

☐ 1. Your Text:
Chapter 7, pages 144-146.

☐ 2. Other Reading Sources:
Vance: Modern College Algebra, Chapter 12, pages 236-237.
Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 185-187.

☐ 3. Audio Tape:
Tape 7.6. "Graphing Logarithmic Functions." The logarithmic function $F(x) = \log_3 x$ will be graphed, and its relationship to the exponential function $f(x) = 3^x$ will be illustrated.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 7.B.

☐ Self Evaluation 7.6

1. Graph the function $f(x) = \log_3 x$, $x > 0$.

☐ OBJECTIVE 7.7

Evaluate and simplify logarithmic expressions by applying the basic properties of logarithms.

Activities 7.7 (We suggest 1 and 3)

☐ 1. Your Text:

Chapter 7, pages 146-149 and pages 158-159. Problems 5-11, pg. 149.

☐ 2. Other Reading Sources:

Eulenberg & Sunko: Introductory Algebra, Chapter 9, pages 228-230.

Vance: Modern College Algebra, Chapter 12, pages 237-239.

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 161-162.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 22, page 213-214, problems 5-9.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 7.B.

☐ Self Evaluation 7.7

1. Express each of the following in terms of $\log_b x$, $\log_b y$, and $\log_b z$.

a) $\log_b\left(\frac{xy}{z}\right)$

b) $\log_b\left(\frac{x^{1/2}}{y^{1/2}z}\right)^3$

c) $\log_b \sqrt[3]{\frac{x^2 y}{z}}$

2. If $\log_b 2 = 0.3010$, $\log_b 3 = 0.4771$, and $\log_b 5 = 0.6990$, find the value of each of the following.

a) $\log_b 10$;

b) $\log_b \sqrt{\frac{4}{5}}$

c) $\log_b \frac{15}{b}$.

3. Express each of the following in terms of a logarithm with base 2.

a) $\log_4 25$:

b) $\log_8 7$;

c) $\log_{\frac{1}{2}} 2N$.

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 7.8

Use the tables of common logarithms to find the log or antilog of a number.

Activities 7.8 (We suggest 1 and 3)

☐ 1. Your Text:
Chapter 7, pages 149-153. Problems 1(a-l),2,3,4,5(a-f),6.

☐ 2. Other Reading Sources:
Eulenberg & Sunko: Introductory Algebra, Chapter 9, pages 230-235.

Vance: Modern College Algebra, Chapter 12, pages 239-242.

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 162-167.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 22, page 215, problems 10, 11(a-g), and 12(a-g).

☐ 4. Audio Tapes:

Tape 7.8. "Using Logarithmic Tables."

The meaning of the terms characteristic and mantissa will be discussed. An introduction to the table of 4-place common logarithms and its use will be given.

☐ 5. Individual Assistance.

☐ 6. Informal Group Sessions.

☐ 7. Lecture 7.B.

☐ Self Evaluation 7.8

1. Use a 4-place table of common logarithms to evaluate the following.

   a) $\log_{10} 3.24$

   b) $\log_{10} 0.0152$

   c) $\log_{10} 7.03 \times 10^5$

2. Find the value of N if:

   a) $\log_{10} N = 0.1875$

   b) $\log_{10} N = 3.9309$

   c) $\log_{10} N = 8.8907 - 10$

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 7.9

Apply logarithms in arithmetic computations.

Activities 7.9 (We suggest 1 and 3)

☐ 1. Your Text:

Chapter 7, pages 156-157. Problems 3,4,5, and 10.

☐ 2. Other Reading Sources:

Eulenberg & Sunko: Introductory Algebra, Chapter 9, pages 235-239.

Vance: Modern College Algebra, Chapter 12, pages 243-244.

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 167-170 and 174-176.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 22, pages 215-217, problems 14-22.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 7.B.

☐ Self Evaluation 7.9

Evaluate the following, using logarithms.

1. $\sqrt[3]{10.7}$,

2. $\dfrac{53 \times 43 \times 10^{-3}}{172}$,

3. $(27.3)^{-1.7}$.

4. Determine the volume of a cylinder with a radius of 2.31 ft. and a height of 4.8 ft.
($V = \pi hr^2$; let $\pi = 3.14$; use logarithms)

Look in the back of this manual for the solutions to this self evaluation. If you solved the problems correctly, you are ready to begin the final objective of this module.

☐ OBJECTIVE 7.10

Solve exponential and logarithmic equations.

Activities 7.10 (We recommend 1 and 3)

☐ 1. Your Text:

Chapter 7, pages 161-162. Work the odd-numbered problems on p. 162.

☐ 2. Other Reading Sources:

Vance: Modern College Algebra, Chapter 12, pages 249-250.

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 8, pages 187-189.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 22, pages 218, problem 27.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 7.B.

☐ Self Evaluation 7.10

Solve the following equations.

1. $4^{2x+1} = 7^{2x-1}$

2. $3^{2x-3} = 27$

3. $\log 4x + \log x = 2$

4. $\log(x^2 - 1) - \log(x + 1) = 3$

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

ASSESSMENT TASK

As soon as you feel you are capable of responding correctly to the questions in all parts of this submodule, ask for the assessment task for Module 7B. It would be well for you to review the self evaluation exercises before taking the assessment. Any materials, tables or supplies required for the assessment other than pencil and paper will be supplied.

WHAT NEXT

After you have taken the assessment over the submodule, you may begin the activities for Module 8. The activities and assessment items relate directly to the objectives which in turn leave no gaps in the total hierarchy. If you need more practice on some part of the module in order to accomplish the objectives, we suggest you choose activities different than your first selection. Your instructor will help you select a path if you will ask.

MODULE 8

TRIGONOMETRIC FUNCTIONS

PREREQUISITE

Module 6

RATIONALE

The name "trigonometry" is derived from Greek and its literal translation is "The measurement of triangles." Its original uses were in the fields of geometry and astronomy. Today, trigonometry means much more than the solving of triangles, and its applications extend to engineering, navigation, the physical sciences, and mathematical analysis.

MODULE ACTIVITIES

Lecture 8 (Check the lecture schedule)

The lecture for Module 8 will begin with a brief introduction to circular functions and their properties. The six basic trigonometric functions will be defined, examined, and graphed.

The inverse relations of the sine and cotangent functions will be graphed and restricted to form inverse functions.

Procedural Options

Option 1. It is suggested that you follow the sequence that follows, utilizing those activities with which you feel most comfortable.

Option 2. You may elect to fulfill the objectives of this module by following the sequence of material as presented in your text in Chapters 4 and 12. You may omit sections 4.5, 4.8, 4.11, and 12.1. It is advisable that you utilize the self evaluations in the following objectives after completing the material in the text.

Approximate Time

This module should take one week to complete.

☐ OBJECTIVE 8.1

Write the coordinates of a point on the unit circle in terms of circular functions.

Activities 8.1 (We suggest 1 and 2A)

☐ 1. Your Text:
Chapter 4, pages 69-74. Problems 1-5, page 72; Problem 1, page 75.

☐ 2. Other Reading Sources:

A. Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 9, pages 190-196.

B. Keedy and Bittinger: Trigonometry - A Programmed Text, Chapter 2, pages 73-83.

☐ 3. Individual Assistance.

☐ 4. Informal Group Sessions.

☐ 5. Lecture 8.

☐ Self Evaluation 8.1

Indicate the coordinates of the following trigonometric points.

1. P(0)

2. P(π)

3. $P(3\pi/2)$

4. $P(-2\pi/3)$

How well did you perform with this self evaluation? Be honest with yourself. The answers are in the Self Evaluation Appendix. If all your answers were correct, go on to Objective 8.2; if not, go over Objective 8.1 a second time and ask for assistance.

☐ OBJECTIVE 8.2

Find values in the range corresponding to specified values in the domain of trigonometric functions, using the trigonometric tables when necessary.

Activities 8.2 (We suggest 1 and 2A)

☐ 1. Your Text:

Chapter 4, page 73 to top of page 79. Problems 1,2,7, and 8, p. 75; Problems 1-2, page 78.

☐ 2. Other Reading Sources:

A. Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 9, pages 207-214.

B. Keedy and Bittinger: Trigonometry - A Programmed Text, (Do not write in this book.) Chapter 2, pages 101-108.

☐ 3. Individual Assistance.

☐ 4. Informal Group Sessions.

☐ 5. Lecture 8.

☐ Self Evaluation 8.2

1. State the domains and ranges of the six elementary trigonometric functions.

2. Indicate the exact value of the following trigonometric functions:

a) sin $\pi/2$

b) cos $(-2\pi/3)$

c) tan $(-\pi/3)$

d) sec $\pi/4$

e) cot $(-\pi/4)$

f) csc $4\pi/3$

3. Using trigonometric tables evaluate the following trigonometric functions:

a) cos $-40°$

b) tan $1°10'$

c) csc $-63°40'$

4. Evaluate using trigonometric tables (use $\pi = 3.14$)

a) sin 1.3

b) cos (-3.48)

c) tan 10

Evaluate your progress. Go on if you are confident.

☐ OBJECTIVE 8.3

Determine the period and amplitude of trigonometric functions.

Activities 8.3 (We suggest 1)

☐ 1. Your Text:
Chapter 4, pages 103-104.

☐ 2. Other Reading Sources:
Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 9, pages 197-202.

☐ 3. Individual Assistance:

Instructors are available to you daily for individual assistance. Come in, even if it is just to say Hello.

☐ 4. Informal Group Sessions.

☐ Self Evaluation 8.3

Indicate the amplitude and period of the following functions:

1. $3 \cos (\pi + 2x)$
2. $2 \sin 1/2x$
3. $-\tan 2x$

Check your answers with those in the Self Evaluation Appendix. If you are having trouble, you may wish to go on to Objective 8.4 and then return to 8.3.

☐ OBJECTIVE 8.4

Sketch the graphs of trigonometric functions.

Activities 8.4 (We suggest 1 and 3)

☐ 1. Your Text:

Chapter 4, pages 102-108. Problems 1-17(odd), pages 107 and 108.

☐ 2. Other Reading Sources:

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 9, pages 197-202.

Keedy and Bittinger: Trigonometry - A Programmed Text, Chapter 2, pages 85-92, 134-140, 151-155.

☐ 3. Solved Problems:

Schaum's: Theory and Problems in Trigonometry, problems 1 and 2 on pages 66 and 67.

☐ 4. Audio Tapes:

Tape 8.4. "Graphing Trigonometric Functions." The functions sine, cosine, secant and tangent are discussed in regard to domain, range, period, and amplitude. Their graphs are sketched and discussed.

☐ 5. Individual Instruction.

☐ 6. Informal Group Sessions.

☐ 7. Lecture 8.

☐ Self Evaluation 8.4

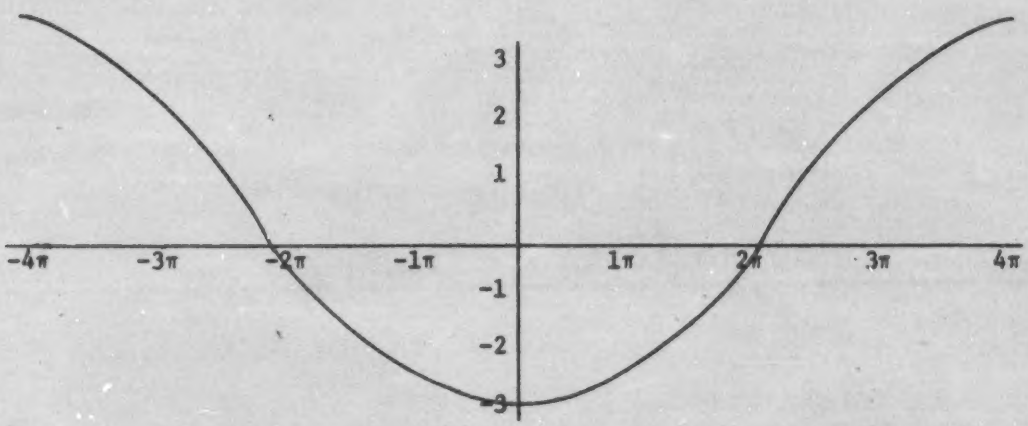
1. Sketch the graphs of the following trigonometric functions:

   a) $2 \cos x$

   b) $\cos 1/2x$

   c) $\cos (2x + \pi/2)$

2. Consider the graph G:



   a) Indicate the amplitude and period of the trigonometric function whose graph is G.

   b) Indicate which of the following trigonometric functions has G as its graph:

1) 3 sin 4x

2) -3 sin (1/4x)

3) -3 cos (1/4x)

4) 3 cos (1/4x)

5) -3 cos 4x

3. Sketch the graph of cos x + 2 sin x, using the graphs of cos x and 2 sin x.

Evaluate your performance on this self evaluation. If you were successful on all three problems, you are ready to proceed to the last objective of this module. If you had trouble with the self-evaluation problems, then try a different set of activities. Perhaps Activities 3 and 5.

☐ OBJECTIVE 8.5

Sketch the graph of a simple inverse trigonometric function.

Activities 8.5 (We suggest 1 and 2B)

☐ 1. Your Text:

Chapter 12, pages 263-270.

☐ 2. Other Reading Sources:

A. Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 11, pages 274-279.

B. Keedy and Bittinger: Trigonometry - A Programmed Text, Chapter 2, pages 200-211.

☐ 3. Audio Tapes:

Tape 8.5. "Inverse Trigonometric Functions." The inverse relations of cosine and tangent are discussed, graphed, and finally restricted to form inverse functions.

☐ 4. Lecture 8.

☐ 5. Individual Assistance.

☐ 6. Informal Group Sessions:

If you haven't tried this activity, you may be missing something.

☐ Self Evaluation 8.5

Sketch the graph of a simple inverse trigonometric function.

1. Sketch the graph of the inverse relation of $f(x) = \sin x$.
2. Restrict the domain of $f(x) = \sin x$ so that it will have an inverse function.

ASSESSMENT TASK

As soon as you feel that you are capable of responding correctly to questions in all parts of this module, make an appointment to complete an assessment task over this material.

WHAT NEXT

We believe that if you performed the activities suggested, you also demonstrated mastery of the material. The converse of that statement is not necessarily true. You may have demonstrated mastery, but did not do the activities. That's fine; no problem. Begin Module 9 with confidence.

You may have performed the suggested activities and yet did not demonstrate mastery. Again, no real problem. Your instructor will help you clear the air if you will ask.

MODULE 9

POLYNOMIAL FUNCTIONS

PREREQUISITE

Competency in Modules 1-7. In particular, competency in Arithmetic of Complex Numbers, Modules 1 and 3. Modules 1 and 3 can be satisfied by Objective 15.1.

RATIONALE

You have met members of this clan before. You conquered linear functions, found zeros of quadratics and even determined the zeros of those polynomial functions of higher degree that could be treated as quadratics. Now you are ready to meet more elusive and wiley members of the clan, those of degree higher than two which are more difficult to factor.

MODULE ACTIVITIES

Lecture 9

The lecture on polynomial functions will consist of selecting a function of the type $f(x) = e^{2x} + 4e^{x} - 1 - 4e^{-x}$ and discuss the rationale for finding the zeros. The discussion will include at least the following: identification of an enabling function and dealing with the enabling function in terms of bounds of zeros; rational zeros; linear approximation of real zeros; complex zeros; multiple zeros and

the graphs of polynomial functions. It will be assumed that you can perform synthetic division and the operations on polynomial functions.

Procedural Options

1. Use the sequence as presented in the text, Chapter 11. The text sequence and sequence of objectives are not the same although the end result should be.

2. Attain competency by traveling through the objectives and activities of each objective.

Average Time

5 days

☐ OBJECTIVE 9.1

Can add and subtract polynomials over the real numbers and write the sum in standard form.

Activities 9.1 (We suggest #1)

☐ 1. Your Text:

Reading Sets 1.12, 1.13, pages 35-39
Problem Sets: 1.4 1, 3, 6, 7, 9, 13, 15, 17, 19, 21
1.5 7, 13, 17, 21, 25, 29
Reading Set 5.5, pages 116,117
Problem Set 5.4, page 117, problems 1, 13, 15, 20

☐ 2. Other Reading Sources:

Vance: Chapter 8, pages 177-191.

Knight and Hoff: Chapter 6, pages 72-109

Cameron: Chapter 13, pages 325-351

Howes, Vernon E.: Pre-Calculus Mathematics, Book I, Algebra. John Wiley and Sons. New York, 1967.

☐ 3. Solved Problems:

Schaum's College Algebra, Chapter 2.

☐ 4. Lecture:

Objective 9.1 will not be a lecture topic.

☐ 5. Individual Assistance.

☐ 6. Informal Group Sessions.

☐ Self Evaluation 9.1

Competency: 100%

1. From the sum of $2x^3 - (1 + \sqrt{3})x - 5$ and $3x^3 - x^4 - 4x^2 + 2\sqrt{3}x$ subtract $4x^3 - 3x + 2x^2 - 3$ and write your result in standard form.
2. $f(x) = x^5 + 4x^3 - 2x^2 + x - 3$, $g(x) = x^3 - 4x^2 + 2x - 3$. Write $(f + g)(x)$ in standard form.

Check your result in the Self Evaluation Appendix.

☐ OBJECTIVE 9.2

Can multiply polynomials over the reals and write the product in standard form.

Activities 9.2 (We suggest #2)

☐ 1. Your Text:

Study Sets 1.14

Problem Set 1.6, page 42, problems 1-24.

☐ 2. Solved Problems:

Schaum's College Algebra, Chapter 2 and 3.

☐ 3. Other Reading Sources:

Vance: pages 171-191.

Howes: pages 508-541.

☐ 4. Individual Assistance in the IPI Center.

☐ 5. Informal Group Sessions.

☐ 6. Lecture:

This topic is assumed in the lecture and will not be presented.

☐ Self Evaluation 9.2

Competency: 100%

1. Write in standard form the result of $(f(x))(g(x))$ if $f(t) = 2t^2 - \sqrt{5}t + 1$ and $g(s) = 3s^3 - \sqrt{5}s$.

2. Write the polynomial in standard form having exactly the following factors:

   a. $x + a$, $ax + b$, $bx^2 + ax + a$

   b. $(x - (1 + i))$, $(x - (1 - i))(x + 5)$

Check your progress before beginning Objective 9.3.

☐ OBJECTIVE 9.3

Can find the quotient and remainder for division of polynomials using synthetic division and demonstrate application of the remainder and factor theorems.

Activities 9.3 (We suggest #1)

☐ 1. Your Text:

Reading Set 11.9, 11.10, 11.11, pages 239-241

Exercise Set 11.8, page 242, problems 1, 3, 5, 7, 8, 9, 11

11.9, page 244, problems 1, 3, 5, 7, 21, 29

☐ 2. Other Reading Sources:

Vance: Section 3.3, pages 42-46 and problems.

College Algebra Text - look up "synthetic division" or "Theory of Equation" or "zeros of Polynomials."

☐ 3. Solved Problems:

Schaum's Outline: pages 187-188.

Text Examples: pages 239-241.

☐ 4. Lecture:

The lecture outline assumes you have this skill prior to the lecture. You may, however, pick up some pointers since synthetic division will be employed.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

☐ Self Evaluation 9.3

Competency: 100%

1. If $f(x) = x^3 - 5x + 2$, find:

   a) $f(-1.5)$ using synthetic division.

   b) $f(-8)$ using synthetic division.

   c) $f(0)$

2. If $x - 2$ is a factor of $f(x) = x^4 - 5x^3 + 8x^2 - 12x + 8$, write the depressed function. If $x - 2$ is not a factor, express $f(x)$ in the equivalent form $f(x) = q(x) + \frac{R}{x - r}$.

3. If $a + 2b$ is a zero of a polynomial $f(x) = x^3 - (a + 2b)x^2 - ax + (a^2 - 2ab)$ find $f(a + 2b)$.

Evaluate your progress by checking your solution.

☐ OBJECTIVE 9.4

Demonstrate application of the fundamental theorem on polynomial functions by finding the zeros of real polynomial functions of degree three or higher.

Activities 9.4 (We suggest #1)

☐ 1. Your Text:

Study Set 11.12, 11.13, 11.14, pages 245-250 and 11.17, pages 256,257

Problem Sets: 11.10, page 248, 1, 3, 5, 11, 13, 16, 17, 19, 23

11.11, pages 250-251, 1, 3, 11, 13, 15, 17
11.13, page 257, 1, 3, 9, 15, 17

☐ 2. Other Reading Sources:

Vance: pages 181-183 and problems.

Knight and Hoff: pages 76-104 (an extensive treatment)

☐ 3. Solved Problems:

Schaum's College Algebra, Chapter 21 (an excellent reference for this topic)

☐ 4. Lecture:

Check the outline of the lecture under module activities, the lecture should be of value to you.

☐ 5. Informal Group Sessions:

Don't overlook the value of learning from others and letting them learn from you.

☐ 6. Individual Assistance.

☐ Self Evaluation 9.4

Competency: 100%

1. Consider the polynomial function $f(x) = 4x^4 - 8x^3 + 7x^2 + 2x - 2$.
   a) List the set of possible rational zeros according to the coefficient of $f(x)$.
   b) Find an upper and a lower bound of real zeros of $f(x)$.
   c) Find all the zeros of $f(x)$.
2. Write the polynomial function in standard form with integral coefficients having $2 - i$ as a zero and -3 as a zero of multiplicity 2.
3. Write the quadratic polynomial in standard form for which the product of the roots is $a^2 + b^2$ and the sum of its roots is $+2a$.
4. Name the roots (zeros) of Problem 3.

Check your progress, recycle any deficiency. The central problem above is 1c, the other problems give conceptual support to this problem or any problem similar.

☐ OBJECTIVE 9.5

Can graph real polynomial functions and approximate the zeros from graphs.

Activities 9.5 (We suggest #1 and #3)

☐ 1. Your Text:

Study Set 11.16, pages 253-255.

Problem Set 11.12, page 255, problems 3, 7, 13.

☐ 2. Other Reading Sources:

Vance: pages 183-191 and related problems.

Schaum's Outline: problem 59, page 203. For Horner's method for approximating irrational zeros, problem 57, page 200.

☐ 3. Lecture.

☐ 4. Informal Group Sessions.

☐ 5. Individual Assistance.

☐ Self Evaluation 9.5

Competency: 100%

1. Sketch the graph of $f(x) = 3x^3 + 4x^2 - 3x - 2$.
2. Use the method of successive approximation or Horner's method to name the positive zero correct to two decimal places.
3. Sketch the characteristics of the following polynomials:

a) constant polynomial b) linear c) quadratic d) cubic e) quartic

Check your solutions in the Self Evaluation Appendix.

ASSESSMENT TASK 9

At this point you should ask for the module assessment if you now know what the objective is, and you feel you have a good chance to demonstrate competency in this module. You will need paper, pencil and graph paper. You will be given an assessment that covers the entire

module styled to enable your instructors to recycle you efficiently if a deficiency occurs.

WHAT NEXT

Four down and two to go to complete the core of level 2. That is, we hope only two to go. If your performance on the assessment was not up to snuff, it should not take long to clear the confusion. Ask your instructor to help you recycle any portion of Module 9 indicated by your responses in the assessment task.

MODULE 10

TRIGONOMETRIC EQUATIONS AND IDENTITIES

PREREQUISITE

Modules 8 and 9.

RATIONALE

You have encountered equations such as $x^2 - 2x + 3 = 0$ called polynomial equations. You have also come in contact with equations such as $\sqrt{x + 1} = 3x$ and $2^x - 5 = 0$ and $2 \log x = 4$, which are examples of irrational, exponential, and logarithmic equations, respectively. In Module 8 you were introduced to the trigonometric functions. In this module you will encounter equations such as $\sin t - \cos 3t = 0$, in which the variable is contained in trigonometric functions. These equations are called trigonometric equations.

There are basicly two types of trigonometric equations, identities and conditional equations. An identity is true for all permissible values of the variable; whereas, a conditional equation is true only for certain values of the variable. You will encounter both types of trigonometric equations in this module.

MODULE ACTIVITIES

Lecture 10

Note 1: Check the Lecture Schedule to see whether Lecture 10 will be presented at an appropriate time for you. If it does not, DON'T WAIT; you should be able to complete this module on your own.

The lecture for Module 10 will begin with an explanation of the terms equation, identity, and conditional equation. The formulas for trigonometric functions of sums, differences, and multiples of numbers will be discussed. Examples of identities and conditional equations will be given and discussed.

Procedural Options

Option 1. It is suggested that you follow the sequence that follows, utilizing those activities with which you feel most comfortable.

Option 2. Cameron presents a very readable and thorough investigation of trigonometric identities and conditional equations in his book, Algebra and Trigonometry With Analytic Geometry. You may elect to read this material (pages 280-301) and work the odd-numbered problems to fulfill the objectives of this module. It is important that you make use of the self evaluations in this module before requesting Assessment Task 10.

Average Time

This module should take no more than one week to complete. If you experience difficulty, request assistance from your instructor.

☐ OBJECTIVE 10.1

Justify that trig equations are identities or conditional equations. Use basic trigonometric identities for a number, formulas for the trigonometric functions of the sum and difference of numbers, and formulas for twice and one-half a number.

Activities 10.1 (We suggest 2A and 4)

☐ 1. Your Text:

Chapter 4, pages 79-82;

Chapter 9, pages 180-188.

☐ 2. Other Reading Sources:

A. Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 11, pages 280-286.

B. Keedy and Bittinger: Trigonometry - A Programmed Text, Chapter 2, pages 110-146.

☐ 3. Solved Problems:

Schaum's: Theory and Problems in Trigonometry, Chapter 10, pages 70-72, problems 9-18; Chapter 11, pages 76-77, problems 5, 6, 7.

☐ 4. Audio Tapes:

Tape 10.1. "Verifying a Trigonometric Identity." A trigonometric identity is presented. Two separate approaches are taken in verifying the identity. Each step is explained in great detail, with emphasis given to the motivation behind each step.

☐ 5. Individual Assistance.

☐ 6. Informal Group Sessions.

☐ 7. Lecture 10.

☐ Self Evaluation 10.1

This is a self-test so that you can show yourself if you really have acquired the skills of this objective. Use it wisely.

1. Using the fact that $15° = 60° - 45°$, find the exact value of $\sin 15°$.

2. Indicate the exact value of $\tan \pi/8$.

3. Indicate the exact value of $\cos 2u$, if it is known that $\pi < u < 3\pi/2$ and $\sin u = -5/13$.

4. Simplify the expression: $\cos(u + v) \cos v + \sin(u + v) \sin v$.

☐ OBJECTIVE 10.2

Solve trigonometric equations using the basic identities and inverse trigonometric functions.

Activities 10.2 (We suggest 2A and 4)

☐ 1. Your Text:

Chapter 9, pages 180-188.

☐ 2. Other Reading Sources:

A. Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 11, pages 286-301.

B. Keedy and Bittinger: Trigonometry - A Programmed Text, Chapter 2, pages 156-158.

☐ 3. Solved Problems:

Schaum's: Theory and Problems in Trigonometry, Chapter 17, pages 126-130, problems 1-19.

☐ 4. Audio Tapes:

Tape 10.2. "Solving a Trigonometric Equation." Two trigonometric equations are presented. Each step in solving the equation is explained in minute detail. The solutions obtained are discussed and tested. At various points in each problem, you will be given the opportunity to proceed on your own.

☐ 5. Individual Assistance.

☐ 6. Informal Small Group Sessions.

☐ 7. Lecture 10.

☐ Self Evaluation 10.2

For each of the following equations, determine whether it is an identity or conditional equation. If it is a conditional equation, solve it for all x. If it is an identity, verify it.

1. $2 \sec x = \tan x + \cot x$

2. $\cos 2x + \cos x + 1 = 0$

3. $\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

Evaluate your progress by checking the solutions in the Self Evaluation Appendix.

ASSESSMENT TASK 10

As soon as you feel that you are capable of performing the tasks required in all parts of this module, make an appointment to complete an assessment task over this material.

WHAT NEXT

Module 10 probably required more time to complete than you first thought. The equations, both conditional and identities, are expressions in the real number system but seem to be surrounded by foggy confusion. Just remember as you begin Module 11, they are <u>real</u> expressions so you can manipulate them under the same rules (properties) you used on those which seemed simpler and more algebraic.

MODULE 11

NUMERICAL TRIGONOMETRY

PREREQUISITES

As indicated by the scheme below (Figure 1) in order to attain the objective, competency in the use of logs, basic trigonometric functions, and identities is required, as well as ability to manipulate algebraic expressions.

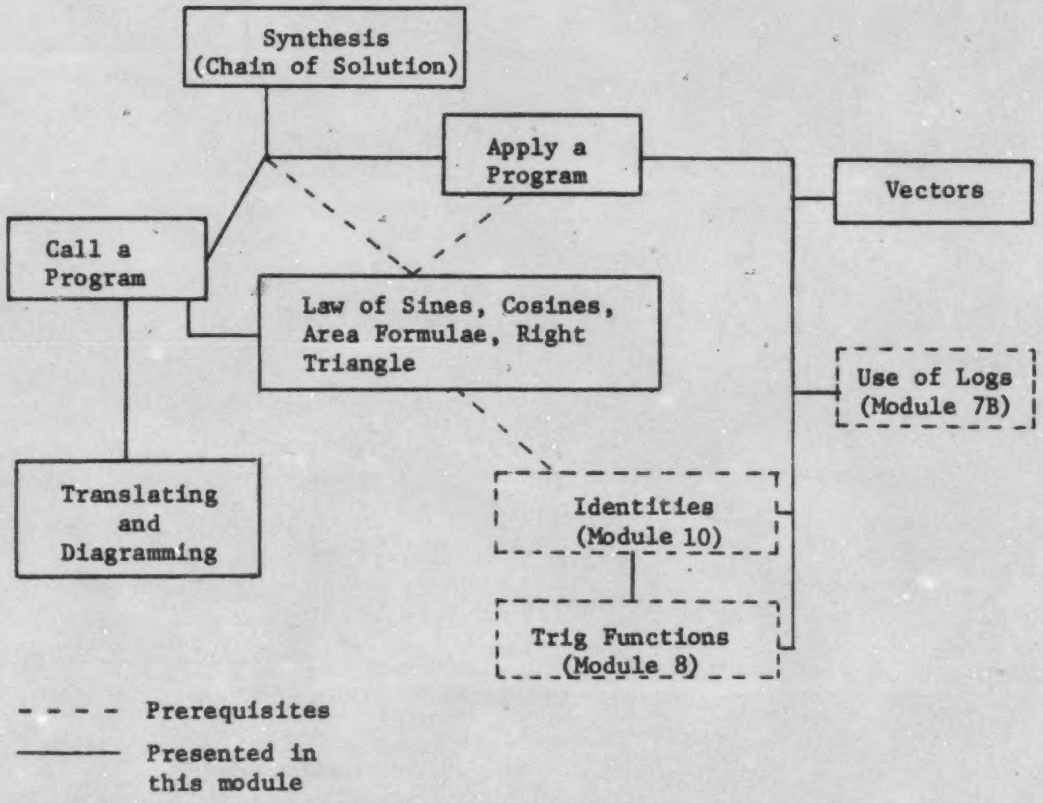


Figure 1

RATIONALE

This unit is essentially an applications unit. The applications involve applying the cosine law and the law of sines or some by-product of these laws to a triangulation problem. Navigation, surveying and problems fitting the physics concept of vector are some of the applications considered.

You have long felt the need for "real world" problems where you can apply numerics rather than literal numbers. Here is your chance, but be aware of the fact that some are not quite "real world." For example, suppose you are to find the distance measure or angle measure to some "real world" problem. The solution to such a problem is usually more easily found by using an instrument rather than a complex computational scheme. Keep in mind that our "practical applications" are over-simplified to teach principles.

MODULE ACTIVITIES

Lecture 11

The outline of the lecture to be presented for this module is as follows:

I. Derivation of Basic Formulae
   - A. Law of Sines
   - B. Law of Cosines
   - C. Area Formula

II. Applications of Formulas
   - A. Classification and Program Selection
     1. Ambiguous case
     2. Use of logs
   - B. Accuracy
   - C. Vectors (As used in Physics)

Procedural Options

The lack of a one-to-one relationship between the sequence of activities by a textual path and objective path shown in Figure 1 exists in this module. You, therefore, have at least two options to achieve the objectives:

Option 1. Study the content if your text (Chapter 8, Robinson) in its sequence and concentrate your efforts on the final objective (11.4) or a similar text section entitled "solutions of triangles," applications. An example is Schaum's Outline Series which is extensive and inclusive. The chapters in the Outline pertain to the objectives of this module are 4, 5, 7, 13, 14, 15.

Option 2. Proceed objective by objective (begin with the activities of 11.1 and work your way through 11.4).

Average Time

Five days in a conventional class are usually set aside for the content of this module.

OBJECTIVE 11.1

Can make a sketch relating to a verbal description of a triangulation problem, label the components according to the data given and identify the requested component.

Activities 11.1 (We suggest 1 and 3 if your option is objective by objective.)

You can probably satisfy this objective without an organized learning packet. Address yourself first to the example diagrams in the test or reference you have chosen. Do the self-evaluation after you become knowledgeable in the following terminology: Components of a right triangle, bearing, heading, angle of elevation, angle of depression, horizontal and vertical components, magnitude and direction of a two dimensional vector.

☐ 1. Your Text:

Chapter 8, example problems.

☐ 2. Other Reading Sources:

Use the sample or example problems that are accompanied by a diagram in the chapter on "Solution of Triangles." The text may be "Law of Sines" or "Law of Cosines," "Practical Applications" or even "Numerical Trigonometry."

☐ 3. Solved Problems:

Schaum's Outline: Chapters 4, 5, 7, 13, 14, 15 (Very good)

☐ 4. Lecture:

The lecture will contain very little diagram construction or sketching.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance in the IPI Center.

☐ Self Evaluation 11.1

Competency: 5 correct of 6

Directions: Sketch a diagram that represents the stated problem. Label the diagram according to the data given and identify the requested component(s) by a variable.

1. A ladder 26 feet long leans against a vertical wall. If the foot of the ladder is on the ground 10 feet from the wall, what is the distance from the ground to the point of contact of the wall and the top of the ladder?

2. Two forces act on a body, one of 20 lbs. and the other of 28 lbs. Their directions make an angle of 72°. Find the magnitude and direction of the resultant of these two forces.

3. A sled is being pulled on level ground by a rope. The tension on the rope is 30 lbs. If the rope makes an angle of 42° with the horizontal, what is the magnitude of the vertical lift on the sled?

☐ 4. A ship sails with a heading of 145° (measured clockwise from the north) and travels at 22 nautical miles per hour. After three hours, the course is changed to 245° and the speed is reduced to 15 nautical miles per hour. What is the distance of the ship from its starting point after 4 hours and what is the heading of the starting point from this position after 4 hours?

☐ 5. The fan belt of an auto wraps around three pulleys the centers of which are the vertices of a triangle with sides 6", 8" and 10". If the pulleys are 3", 4" and 5" diameters, what is the length of the fan belt?

☐ 6. From a look-out tower 90' tall, a ranger observes the angle of depression of the top of a tree is 20°30', while the angle of depression of the base of the tree is 36°40'. If the tree is 215 feet from a point on the ground directly beneath the observer and the base of the tree is in the same horizontal plane as the base of the tower, how high is the tree?

Check your progress by comparing your diagrams with the solution diagrams in the Self Evaluation Appendix. Proceed to Objective 11.2 even if you did not make the required 83%. You may have to pick up this skill en route to Objective 11.4.

☐ OBJECTIVE 11.2

Can call a program (formula or sequence of formulas) that when applied will effect the solution of a triangulation problem, the solution being a component of the triangle or the area of the triangle.

Activities 11.2

"To call a program" means only to select a formula or sequence of formulas to apply on the basis of the given data. If you are completing this module objective by objective (path 2), begin this objective by sorting the three basic formulas, area, law of sines, law of cosines,

to the various cases typed according to the data given. ASA, SSS, SAS, etc. Include in your classification the ambiguous case and the number of solutions according to the data given.

☐ 1. Your Text:

Chapter 8.

☐ 2. Other Reading Sources:

A. Wooten, Beckenbach and Dolciani: Modern Trigonometry, Houghton Mifflin, Chicago, 1966, Chapter 5, Sections 5-7 to 5-10, pages 187-200.

B. Pearson and Allen: Modern Algebra Including Trigonometry, Ginn and Company, Boston, 1966.

☐ 3. Solved Problems:

Schaum's Outline Series: Theory and Problems of Plane Trigonometry, Chapters 4, 5, 13.

☐ 4. Lecture:

Only a portion of the lecture will be addressed to this objective.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

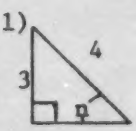
☐ Self Evaluation 11.2

Competency: 9 of 9 part 1; 3 of 3 part 2.

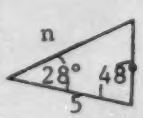
Part 1.

Below are diagrams with data. Write the formula with the data applied so that when the arithmetic is performed, the correct result will be obtained. DO NOT SOLVE.

1)



2)



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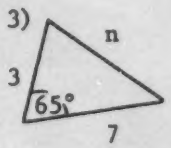
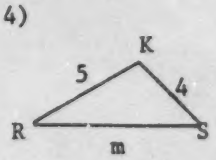
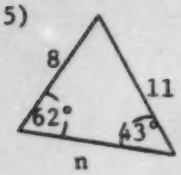


4)

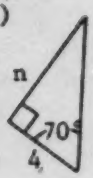


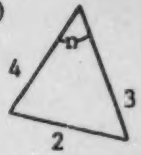
55b

5) 

6)

n represents area

7) 

8) 

9)

Areas 45 units^2

Part 2.

As in part 1, apply a formula with the data inserted that would be used to compute the correct solution of the problem. Define your variable. DO NOT SOLVE.

1. A ship A sails 45 miles n 47°E from a point P. He spots another vessel B located S42W from his new position. B is traveling N22°W at 20 nautical miles per hour. How far east of P is the ship A when he spots B?
2. A regular hexagon (6 sides) is inscribed in a circle of radius 8. What is the radius of the inscribed circle?
3. Two forces of 50 and 75 lbs. act on a body at an angle of 38°. What is the magnitude of the resultant force?

Check your results in the Self Evaluation Appendix.

☐ OBJECTIVE 11.3

Can apply a program to effect the solution of a triangle.

Activities 11.3 (We suggest 1 and 3)

We can demonstrate achievement in decision making on the correct formulas to apply by having completed 11.2. Now the task is arithmetic and use of logs of trig functions.

☐ 1. Your Text:

Any of the problems of the exercise sets of Chapter 8.

☐ 2. Other Reading Sources:

The problems of any Trigonometry text previously cited or any text or treatment of practical applications. You already have use of the basic formulas as demonstrated in Objective 11.2

☐ 3. Solved Problems:

Schaum's Outline: Chapter 13 and 14.

☐ 4. Lecture:

Little time will be devoted to the actual use of logs and the numerics of problem solving. You may need to review Module 7B prior to attending the lecture.

☐ 5. Informal Group Sessions:

Here is a real source of comparison and information you should not overlook. You may get a chip on your shoulder knocked off occassionally, but remember you probably knock off a few chips yourself.

☐ 6. Individual Assistance:

It may be a minor obstruction that your instructor or tutor can clear quickly; seek him out.

☐ Self Evaluation 11.3

Competency: 5 correct of 6

Use the problems of Schaum's Outline in Trigonometry listed below as your self-evaluation.

Page 40, problems 15, 21 and 25.
Page 99, problems 28, 33 and 37.

Check your problems. The solutions are listed by the problem.

☐ OBJECTIVE 11.4

Demonstrate the ability to call and apply a program to solve a triangulation problem which requires synthesizing the solution of more than one triangle.

Activities 11.4

This objective is an extension of 11.2 and 11.3. To accomplish this objective requires the solution of one problem and using this solution as input for the problem requested. There exists many problems of this nature, however, there are too few of these in the texts. The following, however, are suggested.

☐ 1. Your Text:

page 167, problem 14
page 170, problems 22, 23
page 171, problem 28
page 177, problems 17, 18
page 178, problems 9, 10

☐ 2. Solved Problems:

Schaum's Trig. Outline: page 24, problem 25
page 53, problem 24
page 108, problems 25c and 28
page 117, problems 23, 26

☐ 3. Informal Group Sessions.

☐ 4. Lecture.

☐ 5. Individual Assistance.

☐ Self Evaluation 11.4

Competency: 3 of 3.

1. A 15 foot ladder is supported at three points as indicated in the diagram at right, by the ground, by the 6 ft. wall and the vertical wall of a building. Find the height K.

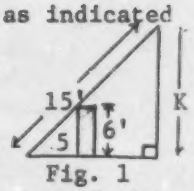


Fig. 1

2. From the top of building A, the angle of elevation of the top of building B 100 feet away is 38° and the angle of depression of the foot of B is 50°. How tall is building B?

3. How far (h) above the base of a can 8 cm in diameter by 20 cm high will the foot of the string touch the wall in the same plane containing the point the string is attached (A), the end points of a diameter of the can (B) & (C) where AB is a straight segment. See Figure 2.

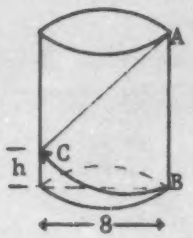


Fig. 2

4. Determine the dimension of the largest rectangle that can be placed inside a trapezoid where the vertices of the rectangle coincide with the vertices of the shorter base and the dimensions are as shown in Figure 3 and the area of the trapezoid is 38.72.

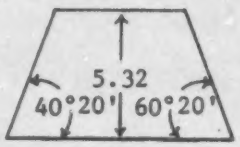


Fig. 3

Check your solutions in the Self Evaluation Appendix.

WHAT NEXT

You are ready for the module assessment. It is constructed in four parts which relate directly to the objectives. Set aside about 50 minutes to complete the assessment. You will have access to trig and log tables. Your performance on the assessment will dictate your next step but think positively, assuming you did well on the assessment. Make your choice of the next module among 12, 14, 15, 16. Your instructor will assist you in making a choice dependent upon your major.

MODULE 12

COMBINATORICS

PREREQUISITE

Competency in Modules 6-11.

RATIONALE

At one time or another, you have certainly had to make a choice involving the arrangement of some set of objects. Have you ever wondered how many distinct choices were possible? As an example, after a baseball coach chooses the starting-nine players for a game he must then choose a batting order. If he realized that there were 362,880 possible batting orders from which to choose, he may have chosen another career.

The branch of mathematics which studies the number of ways that the elements of a set can be arranged or combined into subsets under certain conditions is called combinatorial analysis or combinatorics. Combinatorics assumes an important role in the theory of probability, in statistics, in industry, and in the sciences. In this module we will discuss this important branch of mathematics.

MODULE ACTIVITIES

Lecture 12

Lecture 12 will contain an introduction to the concept of combinatorial analysis and a discussion of the fundamental principle of sequential counting. The terms permutation and combination will be

defined and illustrated by examples. The lecture will conclude with a discussion of the Binomial Theorem and Pascal's Triangle.

Procedural Options

Option 1. You may choose to meet the objectives of this module by relying on the sequence of material as presented in your text (pages 333-343). If you elect this procedure, work most of the problems in the text; and before requesting Assessment Task 12, make use of the self-evaluations in this outline.

Option 2. You may elect to follow the sequence, objective by objective, as presented below.

Average Time - Three days

☐ OBJECTIVE 12.1

Can determine the number of ways two or more events can occur sequentially.

Activities 12.1 (We suggest 1 and 2)

☐ 1. Your Text:
Chapter 15, pages 333-336, problems 1-12, 14, 15.

☐ 2. Other Reading Sources:
Vance: Modern College Algebras, pages 204-206 and problems.

☐ 3. Audio Tape:
Tape 12.1. "The Fundamental Principle of Counting." This brief tape contains a discussion of the process of counting the number of ways two or more events can occur.

☐ 4. Lecture 12.

☐ 5. Individual Assistance.

☐ 6. Informal Group Session:

If you have not made use of this activity previously, you might be missing something.

☐ Self Evaluation 12.1

The student will respond correctly to 3 of the 4 questions.

1. If a baseball team has 6 pitchers and 2 catchers, how many different batteries (combinations of one pitcher and one catcher) are possible?
2. If the first digit cannot be zero, how many five digit numbers can be formed?
3. Simplify $\frac{n(n-1)!}{(n+1)!}$.
4. Evaluate $\frac{7!}{2!3!}$.

Evaluate your progress. Solutions appear in the Self Evaluation Appendix.

☐ OBJECTIVE 12.2

Can identify problems solvable as permutations and find their solutions.

Activities 12.2 (We suggest 1 and 3)

☐ 1. Your Text:

Chapter 15, pages 336-339, problems 1-21 odd.

☐ 2. Other Reading Sources:

Vance: Modern College Algebra, pages 206-209 and problems.

Cameron: Algebra and Trigonometry With Analytic Geometry, pages 356-359 and problems.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, pages 231-235.

☐ 4. Audio Tape:

Tape 12.2. "Permutations and Combinations." This tape defines the terms permutation and combination and illustrates their use in problem solving.

Note: This tape is also used as an activity in Objective 12.3.

☐ 5. Individual Assistance.

☐ 6. Lecture 12.

☐ 7. Informal Group Sessions.

☐ Self Evaluation 12.2

Answer 3 of 4 correctly.

1. Indicate the number of ways 7 books can be arranged on a shelf.
2. Indicate the number of ways 7 books can be arranged on a shelf if two specified books are to occupy the end positions.
3. If a state chooses to represent its license plates with two distinct letters followed by three distinct digits, indicate the number of different plates that are possible.
4. Indicate the number of ways 6 persons can be seated in six chairs at a round table.

Answers appear in the Self Evaluation Appendix.

☐ OBJECTIVE 12.3

Can identify problems solvable as combinations and find their solutions.

Activities 12.3 (We suggest 1 and 3)

☐ 1. Your Text:

Chapter 15, pages 339-341, problems 1-15 odd.

☐ 2. Other Reading Sources:

Vance: Modern College Algebra, pages 210-213, problems 1-19 odd.

Cameron: Algebra and Trigonometry With Analytic Geometry, pages 360-361.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, pages 235-238.

☐ 4. Audio Tape:

Tape 12.2. See Activities 12.2 for description.

☐ 5. Lecture 12.

☐ 6. Individual Assistance.

☐ 7. Informal Group Sessions.

☐ Self Evaluation 12.3

Respond correctly to 3 of 4.

Note: Use of permutations as well as combinations may be necessary.

1. Given 20 points, no three of which are in a straight line, determine the number of straight lines that can be drawn by joining pairs of these points.
2. A committee of 5 members is to be chosen from a group of 10 people. Indicate the number of different committees that are possible.
3. A committee of 5 members is to be chosen, with one member designated as chairman, from a group of 10 people. Indicate the number of different committees that can be selected.

4. Indicate the number of ways to select and arrange a panel of 3 girls and 4 boys from a group of 7 girls and 8 boys. Leave your answer in terms of the notations c(n,r) and/or P(n,r).

☐ OBJECTIVE 12.4

Can compute the coefficient of the r^{th} term of $(ax + by)^n$, $1 \leq r \leq n$, and expand a binomial such as $(2x - 3y^2)^5$.

Activities 12.4 (We suggest 1 and 2B)

☐ 1. Your Text:
Chapter 15, pages 341-343.

☐ 2. Other Reading Sources:
A. Vance: Modern College Algebra, pages 213-216 and problems.

B. Cameron: Algebra and Trigonometry With Analytic Geometry, pages 128-130 and problems on pages 132-133.

☐ 3. Solved Problems:
Schaum: Theory and Problems in College Algebra, pages 156-159.

☐ 4. Lecture 12.

☐ 5. Individual Assistance.

☐ 6. Informal Small Group Sessions.

☐ Self Evaluation 12.4

Respond correctly to 3 of 4.

1. Write the expansion of $(x - y)^5$.

2. In the expansion of $(2x^2 + y)^3$ determine the coefficient of the term involving x^2y^2.

3. Write the expansion of $(2x - 5)^4$ and simplify.

4. In the expansion of $(x - y^2)^7$ write all terms whose coefficients are equal to $C(7,3)$.

Evaluate your progress by checking the solution in the Self Evaluation Appendix.

ASSESSMENT TASK 12

When you have accomplished the objectives of this module, request to take Assessment Task 12.

WHAT NEXT

It is suggested that you begin Module 13 after completing Assessment Task 12 with the required competency.

MODULE 13

PROBABILITY

PREREQUISITE

Module 12

RATIONALE

The following introduction by J. Vincent Robison is quoted from page 343 of your text.

> The theory of probability had its origin in games of chance (gambling). The foundations of the theory were laid in the seventeenth century by the mathematicians Pascal and Fermat. Since that time the theory of probability has increased in importance until today it is vital to our national survival. The insurance industry, one of the largest businesses in the world, is based on the ability to determine the likelihood of the occurrence of certain events. Probability methods are applied in such fields as quantum mechanics, the design of experiments, the interpretation of data, traffic control, the theory of strategy, the allocation of equipment. A knowledge of the theory is vital in analyzing problems in communications and control, quantum mechanics, and kinetic theory.
>
> Probabilities are associated with the outcomes of experiments. In each experiment there is a set of all conceivable outcomes, and we are interested in the chances that particular outcomes will result from the experiment.

MODULE ACTIVITIES

Lecture 13 (Check the Lecture Schedule)

The lecture for Module 13 will begin with a brief introduction to the theory of probability and the basic terms that will be used

throughout this module. The concepts of the probability of simple events, the probability of compound events, conditional probability, and the binomial distribution will be discussed and illustrated in the solutions of particular problems.

Procedural Options

Option 1. It is recommended that you follow the sequence of objectives as presented in the following pages. Use those activities with which you feel most comfortable and be honest with yourself in the self evaluations.

Option 2. The material in your text (pages 343-355) is well organized and may suffice as your principal source in meeting the objectives of this module. If you elect to use your text in this capacity, it is vital that you work the majority of problems encountered and that you make use of the self evaluations before requesting Assessment Task 13.

Average Time

4 days

☐ OBJECTIVE 13.1

Compute the probability of a single event. Become familiar with the basic terms related to probability theory: sample space, event, outcome, success, equally likely events, and probability function.

Activities 13.1 (We suggest 1 and 3)

☐ 1. Your Text:
Pages 343-349. It is extremely important to read and work many of the problems appearing on page 349.

☐ 2. Other Reading Sources:
Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 14, pages 363-369.

☐ 3. Solved Problems:

Schaum: Theory and Problems of College Algebra, Chapter 25, pages 243-245, problems 1a, c, d, 3, 8, 9a, b, c, d, 10a.

☐ 4. Individual Assistance:

Your instructors are available daily for individual help. Come by and show your face.

☐ 5. Informal Small Group Sessions:

The project room is a good place to study with other students. If you haven't made use of this valuable activity, you may be missing something.

☐ 6. Lecture 13.

☐ Self Evaluation 13.1

Competency: All five responses must be correct.

Ten cards, numbered from 1 to 10, are placed in a box.

1. If an experiment consists of drawing one card at random and recording its number, indicate the sample space of this experiment. _____.

2. The probability of drawing a seven in one attempt is _____.

3. The probability of drawing a number less than seven is _____.

4. The probability of drawing a number greater than or equal to seven is _____.

5. If two numbers are drawn (without replacement) the probability that they are both even numbers is _____.

Evaluate your progress by comparing your responses to those appearing in the Self Evaluation Appendix. If all of your responses were correct, then begin work in Objective 13.2. Otherwise, review the previous material.

☐ OBJECTIVE 13.2

Compute probabilities for compound events. In particular, $P(A \cup B)$, $P(A \cap B)$, and $P(A')$.

Activities 13.2 (We recommend 1, 3 and 4)

☐ 1. Your Text:

Pages 347-352. Problems 1,2,3,15 and 18 on pages 354 and 355.

☐ 2. Other Reading Sources:

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 14, pages 371-372.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 25, pages 243-244, problems 2, 4, 5, 6, 12, 14.

☐ 4. An Experiment:

Material needed: one die, a pencil, and paper. On the paper, list the numbers 1, 2, 3, 4, 5, 6 in a vertical column.

Now toss the die and record by a tally mark the number which appears on the die.

Repeat this routine 50 times.

Let N_1 be the number of times a "one" was showing on the die; let N_2 be the number of times a "two" has appeared.

Compare the numbers $\frac{N_1}{50}$ and $\frac{N_2}{50}$ with 1/6, the probability that a particular number will appear.

Compare the number $\frac{N_1 + N_2}{50}$ with 1/3, the probability that a "one" or "two" will appear.

☐ 5. Individual Assistance.

☐ 6. Informal Small Group Sessions.

☐ 7. Lecture 13.

Self Evaluation 13.2

Competency: 4 correct responses

One card is drawn from a deck of 52 bridge cards.

1. The probability that the card is a seven or a king is _____.

2. The probability that the card is a seven or a heart is _____.

3. The probability that the card is not a seven or a heart is _____.

4. The probability that the card is red or a seven is _____.

Evaluate your responses and be honest with yourself before advancing to Objective 13.3.

☐ OBJECTIVE 13.3

Compute conditional probabilities and distinguish between independent and dependent events.

Activities 13.3 (We suggest 1 and 3)

☐ 1. Your Text:
Pages 351-353. Look at Problems 19-21 on page 355.

☐ 2. Other Reading Sources:
Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 14, pages 373-378.

☐ 3. Solved Problems:
Schaum's: Theory and Problems of College Algebra, page 246, #13 and page 248, #18.

☐ 4. Individual Assistance.

☐ 5. Informal Group Sessions.

☐ 6. Lecture 13.

☐ Self Evaluation 13.3

Competency: 4 correct responses.

Two cards are drawn from a deck of 52 bridge cards.

1. It is known that one of the cards is an ace. The probability that both cards are aces is _____.
2. It is known that one of the cards is not an ace. The probability that neither card is an ace is _____.
3. It is known that one of the cards is red. The probability that both cards are red is _____.
4. It is known that one of the cards is red. The probability that both cards are hearts is _____.

Don't kid yourself. If you are having trouble, come in for HELP. Conditional probabilities often trip up even sure-footed students.

☐ OBJECTIVE 13.4

Determine the probability of an event occurring k times in n independent trials, using the binomial distribution.

Activities 13.4 (We suggest 1 or 2)

☐ 1. Your Text:

Pages 353-355. Pay close attention to problems 10, 12, 13, 14, and 18.

☐ 2. Other Reading Sources:

Cameron: Algebra and Trigonometry With Analytic Geometry, Chapter 14, pages 379-380.

☐ 3. Solved Problems:

Schaum's: Theory and Problems of College Algebra, Chapter 25, page 244, #8; pages 249-250, 22-26.

☐ 4. Lecture 13.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

☐ Self Evaluation 13.4

Competency: 5 correct responses.

Four dice are thrown.

1. The probability that all four dice show "two's" is _____.
2. The probability that all four dice show the same number is _____.
3. The probability that exactly three of the dice show a "two" is _____.
4. The probability that at least three are "two's" is _____.
5. The probability that two are "two's" and the other two differ from these and from each other is _____.

ASSESSMENT TASK 13

As soon as you feel that you are capable of responding correctly to questions in all parts of this module, make an appointment to complete an assessment task over this material.

WHAT NEXT

If you did not successfully complete all parts of the assessment task, you will need to direct your efforts toward those objectives in which a deficiency was observed. Take full advantage of your instructors and the material available.

If you were successful in all parts of the assessment task for this module, you may begin any of the remaining modules.

MODULE 14

LINEAR ALGEBRA

PREREQUISITES

Competency in Module 4 (Linear Equations, Inequalities and Linear Functions) and core Modules 6-11.

RATIONALE

There are many problems in business, economics, engineering and psychology which can be stated by means of a system of linear equations. Such a system might contain over 500 equations. It is obvious that solving such systems by the methods of addition-subtraction, substitution, etc., employed in previous modules or courses is impractical. There arises the need for a more powerful method. One such method is based on the concept of a matrix in which the mathematical object is represented in a rectangular array of numbers. With computers available to perform certain defined operations on matrices based on the theory of linear equations, a rationale is established.

This introduction to linear algebra will end with a mechanical method for solving systems called Cramer's rule. Cramer's rule leads to the concept of a determinant which is an indispensable tool in more advanced work. We shall also solve systems via matrices which require proficiency in performing operations on matrices and vectors which are special cases of matrices.

MODULE ACTIVITIES

Lecture 14

The lecture outlines below is ambitious for a fifty minute period. The topics covered will be in the sequence listed. Parts IV through VI are straightforward and rather mechanical and can readily be learned using the text.

I. Vector Operation
   sums, scalar products and dot products

II. Matrix Row Operations for System Solution

III. Algebra of Matrices
   operations, properties of operation

IV. The Determinant of a Square Matrix
   evaluation of a determinant

V. Properties of Determinants

VI. System Solution by Determinants

Procedural Options

Option 1. Use the sequence that follows using those activities with which you feel most comfortable.

Option 2. Follow the sequence outlined in your text of Chapter 13 and the self evaluation exercises of the objectives that follow.

Average Time - Five days.

☐ OBJECTIVE 14.1

Can perform the following operations on vectors where defined when the vector is represented as ordered n-triples by row or by column: name the sum, scalar product, dot product, length or magnitude and express a vector in terms of a unit vector.

Activities 14.1 (We suggest 1 and 5)

☐ 1. Your Text:

Section 8.3, pages 171-173

Exercise Set 8.3, pages 174-175, problems 14-18.

Note: The vectors presented in this section are only ordered pairs. (The sums, differences and scalar products on vectors as n-triples requires a small extension. The dot product of two vectors is not treated in your text.)

☐ 2. Other Reading Sources:

Vance: Section 13.1, 13.2, pages 251-260; problems 2, 3, 10, 12, 13, 18, 23-30, page 256; problems 1, 7, 13, 16, 19, 27 (the product defined is not the dot product)

Henderson, Usiskin, Zaring: Chapter 10 and problems of Chapter 10.

☐ 3. Individual Assistance.

☐ 4. Informal Group Sessions.

☐ 5. Lecture 14.

☐ Self Evaluation 14.1

Competency 6 of 6.

Let $u_1 = (1, -\sqrt{3}, 4)$

$u_2 = (1, -\frac{1}{2}, 2\sqrt{3})$

$u_3 = (0, 1, 0)$

$u_4 = (\sqrt{3}, \frac{3}{2})$

Find, if the operation is defined. If the operation is not defined, indicate by the remark "not defined".

1. $u_1 + u_2$
2. $5 - u_1 \cdot u_2$
3. $2u_1 - (u_2 \cdot u_3)(u_3)$
4. $u_3 \cdot (u_1 + 4u_2)$

5. $\| u_4 \|$ (length or norm)

6. Express u_4 as the magnitude of the vector times a unit vector.

☐ OBJECTIVE 14.2

A. Can test the equals relation on matrices.
B. Can perform correctly the following operations on matrices where the operation is defined: name the sum, product and the inverse of a square matrix (n = 2).

Activities 14.2 (We suggest 1 and 2A)

☐ 1. Your Text:

Section 13.5, pages 285-290
Section 13.7, pages 295-300
Exercise Set 13.4, pages 290-291
Exercise Set 13.6, pages 300-302

☐ 2. Other Reading Sources:

A. Vance: Section 7.1-7.3, pages 147-157
Page 149, problems 1, 4, 9, 23, 25, 27, 28
Page 152, problems 1, 5, 9, 11, 15, 21
Page 157, problems 1, 3, 5, 7

B. Cameron: Section 6.1-6.3, pages 86-97
Page 91, problems 1-20
Page 96, problems 9-16, 23

☐ 3. Individual Assistance.

☐ 4. Informal Group Session.

☐ 5. Lecture 14.

☐ Self Evaluation 14.2

Competency: 6 correct of 7.

Let: $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}$

$D = \begin{pmatrix} 1 & 0 \\ 1 & -1/2 \\ 1/3 & -1 \end{pmatrix}$ $E = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$

Perform, if defined. If not defined, indicate.

1. $A + B$
2. DC
3. $CD - B$
4. B^{-1}
5. A^2
6. If $g(x) = x^2 + 3x - 10I$, find $g(A)$.
7. Which of the following cannot be performed?

   a) DC b) DE c) D^2 d) CD + E e) ED

Check your progress using the notes and solutions in the Self Evaluation Appendix.

☐ OBJECTIVE 14.3

Solve a system of linear equations using matrices.

Activities 14.3 (We suggest 2A and 2B)

☐ 1. Your Text:

Problems 27, 28, page 302.

☐ 2. Other Reading Sources:

A. Vance: Page 158, problems 11, 12, 13, 15

B. Cameron: Section 6.3, pages 92-97 and problems, page 96.

☐ 3. Individual Assistance.

☐ 4. Informal Group Sessions.

☐ 5. Lecture 14.

☐ Self Evaluation 14.3

Competency: 5 correct of 6.

Consider the systems:

S_1: $\begin{matrix} 3x - 5y = 4 \\ 2x + 3y = 7 \end{matrix}$ S_2: $\begin{matrix} 2x + y - z = 11 \\ 2y + z = 0 \\ 5x + 2y - 3z = 11 \end{matrix}$ S_3: $\begin{pmatrix} 1 & -1 & 0 & 2 \\ 3 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 1 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \\ -1 \end{pmatrix}$

1. Express system S_3 as an equivalent set of linear equations.
2. Express system S_1 as a matrix equation.
3. Name the inverse of the coefficient matrix of S_1.
4. Employ the results of Problem 2 and Problem 3 to solve system S_1 (be able to show).
5. If B is the coefficient matrix of S_2 and $B^{-1} = \begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & 4 \end{pmatrix}$ Solve S_2. (be able to show)
6. Solve S_1 by the method of matrix elimination (use row reduction on the augmented matrix until the coefficient matrix is transformed to the multiplicative identity matrix. Be able to show.

☐ OBJECTIVE 14.4

Solve a system of equations using Cramer's rule.

Activities 14.4 (We suggest #1)

☐ 1. Your Text:

Reading Sets 13.8-13.10, page 311

Exercise Set 13.7, page 305, problems 3, 7, 13, 19, 23, 25, 27

Exercise Set 13.8, page 311, problems 1, 4, 5, 9, 13

☐ 2. Other Reading Sources:

Vance: Articles 7-4 through 7-7, pages 159-176
Page 160, problems 5, 11, 15
Page 162, problems 1, 3, 13, 16, 18
Page 167, problems 1, 3, 5, 7
Page 173, problem 3
Page 175, problems 9, 11

Note: Cramer's rule is not included.

Cameron: Articles 6.4-6.11, pages 97-119 and problems related.

☐ 3. Solved Problems:

Schaum's Outline: College Algebra, Chapters 26, 27.

Caution: the method of evaluating a third order determinant presented is not useful for 4th or higher order determinants.

☐ Self Evaluation 14.4

Competency: 100%

Use the following systems for problems 1-4:

$$S_1: \begin{array}{l} 3x - 5y - 2z = 20 \\ x + y + z = 3 \\ 8x + 4y + 3z = 27 \end{array} \qquad S_2: \begin{array}{l} x + y + z + 2w = 1 \\ 2x + 3y + 3z + w = 0 \\ 4x + y + 5z - w = 2 \\ x - y - z - w = 0 \end{array}$$

1. Write the determinant of the coefficient matrix of S_2.
2. Evaluate the determinant of the coefficient matrix of S_1.
3. Determine the solution for S_1 employing Cramer's Rule.
4. If the value of the determinant of the coefficient matrix of S_2 is 36, find the value of y.

Check your progress by comparing your answers with those in the Self Evaluation Appendix.

ASSESSMENT TASK 14

If you have completed sufficient activities to enable you to perform well on the self evaluation exercises, you are ready for the assessment instrument on this module. You will need only pencil and paper to take the assessment task.

WHAT NEXT

Three alternatives exist:

1. Recycling because performance level on Assessment 14 not sufficient to continue.
2. Stop and take the grade you have.
3. Choose your next module from 12, 13, 15 or 16. Module 13 has Module 12 as a prerequisite.

We hope you choose the third alternative.

MODULE 15

COMPLEX NUMBERS

PREREQUISITES

Core Modules 6-11 or equivalent.

RATIONALE

You have already found that the real number system is an unsatisfactory backdrop for solutions to quadratic equations. Some quadratics with real coefficients have real roots while others lack this characteristic. The efforts to observe this apparent discriminatory behavior, a kind of algebraic liberation movement, produced a better microscope. In fact, now the roots of every algebraic equation, regardless of its degree, can be isolated. The result of the efforts was the complex number system and the solutions of algebraic equations is but one of the achievements of this system.

You should consider the field properties (closure, commutative, etc.) as you perform the activities that follow. The characteristics of the system is not, however, main thrust of this module. You should be able to perform basic operations with complex numbers and change forms of these numbers. You should also recognize that the real number system, the system you have studied so long is a prime ingredient but is a mere drop in the bucket, a small (small not appropriate in infinite sets) sub-field of the complex number system.

MODULE ACTIVITIES

Lecture 15

The lecture outline listed below is rather ambitious for a fifty minute period. It will be given in the sequence listed.

Objectives 15.1 & 15.2 I. Complex Number Forms

rectangular, ordered pair, polar, matrix and geometric representation of complex numbers

Objectives 15.2 & 15.3 II. Fundamental Operations With Complex Numbers

addition, subtraction, multiplication, division

Objective 15.3 III. DeMorvies Theorem

powers and roots of numbers

Procedural Option

You may elect:

Option 1. Proceed by the sequence of objectives and activities that follow.

Option 2. Study Chapter 10 of your text or similar treatment in a college algebra text.

Average Time - Four days.

OBJECTIVE 15.1

Given two complex numbers in any of the forms $a + bi$, $a + \sqrt{b}$, $b < 0$, or (a,b), can test the equals relation, perform basic operations on the numbers and represent the numbers on the complex plane.

Activities 15.1 (We suggest #1 and 2C)

The objective stated is a combination of Objectives 1.4 and 3.3 of Level 1 and is written here for your convenience. It is important that you be proficient in performing each primary operation on a pair of complex numbers written in the forms $a + bi$, (a,b) or $a + \sqrt{b}$, $b < 0$.

☐ 1. Your Text:

Sections 10.1-10.5, pages 201-209

Exercise Set 10.1, pages 207-208

Be sure you can perform each of the operations on complex numbers written in each of the forms $a + bi$, (a,b) or $a + \sqrt{-c}$, $c > 0$.

☐ 2. Other Reading Sources:

A. Vance: Section 13-3, pages 261-265 and related problems.

B. Dubisch and Howes, Intermediate Algebra, 2nd Edition, Wiley and Sons, New York, 1969. Chapter 11, pages 298-320. Here is a discussion showing representation of a complex number by a matrix. Complex numbers as ordered pairs are not discussed.

C. Howes: Section 3.45-3.66, pages 120-142.

☐ 3. Solved Problems:

Schaum's Outline: College Algebra, Chapter 8, pages 63-66. Complex numbers as ordered pairs not presented. However, if you will use the definition: $a + bi = (a,b)$, you can make the necessary extension.

☐ 4. Individual Assistance.

☐ 5. Informal Group Work.

☐ 6. Lecture 15:

The lecture will provide only a short review addressed to this objective.

☐ Self Evaluation 15.1

Competency: 7 correct of 7

Perform the indicated operation and express the result in form expressed in the problem.

1. $(4 - \sqrt{-3}) + (2,-3\sqrt{-3})$
2. $(2 + 3i) - 3 + 3 - 5i + 2i$
3. $(3,-1) - (-2,-3)$
4. $(1 - 3i) \div (2 + i\sqrt{5})$
5. $(2 - 6i)(8 + 4i)i$
6. $\dfrac{(3 + i)^2}{5 - i}$
7. If $(x - 3iy)(i + 4) = 10 - 23i$, find real value of x and y.
8. Represent graphically the complex numbers: A(1,-2), B(3 - 2i), C(-2i), D(3).

Check your solutions. Individual assistance is available if you are having problems.

☐ OBJECTIVE 15.2

Can change one form of a complex number to any other form when the forms are $a + bi$; (a,b); $a + \sqrt{b}$, $b < 0$; $r(\cos\theta + i \sin\theta)$ or $r \text{ cis } \theta$ and locate the point in the complex plane.

Activities 15.2 (We suggest #1)

You are familiar with the first three forms named having completed 15.1. The activities that follow should enable you to transform from one form to another. The main emphasis of these activities is not change of form, rather working with polar form or trigonometric form. This form of which $2(\cos 40 + i \sin 40)$ is an example and is of ten abbreviated to $2 \text{ cis } 40$ (cos i sin) is a more convenient form in performing certain operations.

☐ 1. Your Text:

Study Sets 10.5-10.7, pages 208-213. Randomly select several problems of each type from Exercise Sets 10.2 and 10.3.

☐ 2. Other Reading Sources:

Vance: Section 13.5, pages 266-269
Problems 1-7, pages 269-270

Dubisch and Howes: Chapter 11

☐ 3. Solved Problems:

Schaum's Outline Series: College Algebra, Chapter 20, pages 172-181. Problem 2, page 176 and problems 14, 15, 16, page 180.

Schaum's Outline Series: Trigonometry, Chapter 18, pages 134-145. Problems 1-10, 12, 13, page 138 and problem 23, page 144.

☐ 4. Lecture 15.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

☐ Self Evaluation 15.2

Competency: 100% except optional problems.

You may use tables of trigonometric functions.

1. Complete the chart below by expressing the given complex number in the other named forms.

	Ordered Pair	Rectangular	Polar or Trigonometric	Misc.	Matrix
A			$2\cos(-150) + i\sin(-150)$		(optional)
B	(0,3)				(optional)
C		$2.12 - 2.12i$			(optional)
D			$-5(\cos 2\pi/3 + i \sin 2\pi/3)$		(optional)
E				$\frac{1}{2 - 3i}$	(optional)

2. Name the modulus, the amplitude as an angle between $0°$ and $360°$ and locate the number in the complex plane.

   A. $2(\cos 40 + i \sin 40)$

   B. $-3 \text{ cis } 26°$

   C. $-3 + 5i$

   D. $(\sqrt{3}, -1)$

3. Show the relation below is an identity:

   $$[3(\cos 30° + i \sin 30°)][2(\cos 90° + i \sin 90°)] = 6(\cos 120° + i \sin 120°)$$

4. (Optional) Show that if we restrict ourselves to 2×2 matrices which represent complex numbers, multiplication is commutative, ie. $A \cdot B = B \cdot A$.

5. Solve for a and θ

   $$2a(\cos \theta + i \sin \theta) = 5 + 2i$$

Check your solutions and evaluate your progress.

☐ OBJECTIVE 15.3

Can perform multiplication and division on complex numbers and find powers and roots of complex numbers.

Activities 15.3 (We suggest #1)

☐ 1. Your Text:

Study Sets 10.7-10.8, pages 211-217
Exercise Set 10.3, page 213, odd numbered problems.
Exercise Set 10.4, page 217, problems 3, 5, 9, 17, 21, 23, 25 29, 31, 33

☐ 2. Other Reading Sources:

Vance: Section 13-5, 13-6, pages 266-274
Page 270, problems 3, 4, 5
Page 274, a problem of each type.

Look for DeMoivre's Theorem in any college algebra or trigonometry text.

☐ 3. Solved Problems:

Schaum's Outline Series: Algebra, Chapter 20, pages 172-181. We suggest the supplementary problems of page 180.

Schaum's Outline Series: Trigonometry, Chapter 18, pages 134-143.

☐ 4. Lecture 15.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

☐ Self Evaluation 15.3

Competency: 7 of 8

Evaluate and express final result in $a + bi$ form.

1. $2[\cos 240° + i \sin 240°] \cdot 7[\cos(-120°) + i \sin(-120°)]$

2. $(-21)[\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}] \div (-7)[\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}]$

3. $(2 + 2i\sqrt{3})(-\frac{1}{2} + \frac{1}{2} i) \div (1 - i)$

4. $(\cos 5° + i \sin 5°)^{12}$

5. $(\sqrt{3} + i\sqrt{3})^{10}$

6. Find four distinct fourth roots of 1.

7. Find the solution set and express in $a + bi$ form:

$$x^2 - (2 - i)x + 2i = 0$$

8. Plot the three distinct cube roots of $2i$.

Check your solutions in the Self Evaluation Appendix.

ASSESSMENT TASK 15

You might review the Self Evaluation exercises before asking for Assessment Task 15. A pencil is all you will need; graph paper will be supplied.

WHAT NEXT

We hope you did well on the assessment. In that event choose another module. If this module (15) happens to be your last, we congratulate you. You may enroll in the next course with the confidence that you have no pre-requisite deficiences. Visit your instructor in the PIPI Center and tell him what was most valuable to you in this enterprise.

MODULE 16

SEQUENCES

PREREQUISITES

Competency in the core modules (6-11).

RATIONALE

Sequences are ordered sets of real numbers, finite or infinite. A sequence is usually considered to be the range of a function whose domain is the set of positive integers or a finite subset of the positive integers.

Since the domain of a sequence function contains positive integers, sequences have applications in problems where it is possible to count input information. Problems involving the number of years, how many times, how far, etc. lend themselves to the use of sequences.

MODULE ACTIVITIES

Lecture 16

One lecture is scheduled for this unit. The outline of the lecture is sketched below. Emphasis in terms of time will be given in reverse order to the listing shown; that is, the most time allotted to any topic will be to part IV.

I. Definitions and terminology sequences, series

(Objectives 16.1-16.3) II. Types of sequences and sum formulas geometric, arithmetic

(Objective 16.4) III. General sequences, specific terms, n^{th} or general term

(Objective 16.5) IV. Mathematical induction, proofs of sum formulas, proof of sum of n terms for general sequences.

Procedural Options

The end of the level is in sight. You have weaved your way through the module with sensations ranging from frustration and anger to confidence. There have been times when you beheld the beauty of mathematics and times when you could see nothing but weeds. Though the general direction of the tour was planned for you, you were free to linger at any time at any place, choosing your own path.

By now you have found a style, or a path of maximum efficiency for you. You should apply this style, if it is honest, to this module and to future courses regardless of the mode of instruction.

By now you realize mathematics "cannot be taught" but "must be learned;" that mathematics is not a spectator sport but is a participants game. This module should emphasize your role of participation and intuition.

Apply your style and finish the race.

Average Time

4 days

☐ OBJECTIVE 16.1

The student will demonstrate the ability to calculate specific terms of a sequence - say the first four, the 41st, the 29th, etc. - given the n^{th} term.

Activities 16.1 (We suggest #1)

☐ 1. Your Text:
Study Set 14.1, pages 314-317
Exercise Set 14.1, page 317

Note: the determination of a specified term is discussed in example three, page 316, but is not requested in the exercise set following. You should verify that the 20th term and the 39th term of the sequence described in problem 9 of page 317 are 1/420 and 1/478, respectively.

☐ 2. Solved Problems:

College Algebra text example problems in chapters labeled "Sequences," "Series," or "Progressions."

Schaum's Outline: College Algebra; Definitions of page 280, Reading material, page 284-285; problems 1-6 and verify in problem 2c of page 284 that the twelfth term and the twenty-seventh term of the series are $\frac{-\sqrt{12}}{13} = \frac{-2\sqrt{3}}{13}$ and $\frac{+\sqrt{27}}{28} = \frac{+3\sqrt{3}}{28}$ respectively.

☐ 3. Other Reading Sources:

Henderson, Usiskin, Zaring, Precalculus Mathematics. McGraw Hill, 1971. Chapter 13.

Pearson, Allen, Modern Algebra, Book Two. Ginn and Company, 1966.

Any title or topic including "Sequences, Series or Progressions" found in College Algebra texts available in the library.

☐ 4. Lecture.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

☐ Self Evaluation 16.1

Competency: 4 correct of 4

1. Find the first four terms of

a) $\left\{\frac{n(n-3)}{2}\right\}$

b) $\{(-1)^n + 3\}$

2. Find the 10th and 15th term of these sequences:

   a) $(3n - 2)a + 4(7 - n)b$

   b) $7n - 20$

3. Write a recursive formula for $\{4, 7, 11, 16, \ldots\}$

Check your progress and recycle or continue to 16.2 according to your answers of the self evaluation.

☐ OBJECTIVE 16.2

Given a list of sequences, the student can identify the arithmetic sequences, determine their common difference and calculate the sum of the first n terms.

Activities 16.2 (We suggest #1 and #2)

Keep in mind the three verbs of the objective as you trek through the module. Calculating correctly the sum, and determining the common difference depend on identifying the sequence as arithmetic.

☐ 1. Your Text:
   Section 14.3, pages 321-322
   Exercise Set 14.3, page 324, problems 1-20

☐ 2. Solved Problems:
   Schaum's Outline: College Algebra. Read "Arithmetic Progressions", page 140. Practice problems 1-24, pages 142-143.

☐ 3. Other Reading Sources:
   Same as 16.1.

☐ 4. Lecture.

☐ 5. Informal Group Sessions.

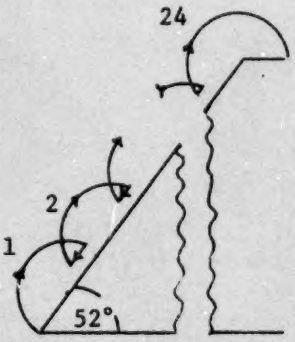
☐ 6. Individual Assistance.

☐Self Evaluation 16.2

Competency: 100%

Determine which of the following sequences are arithmetic progressions and find the common difference d, S_{10} and the 20th term for each such progression for problems 1-7.

1. 2, 5, 8, 11, ...
2. 3, 5, 8, 12, ...
3. 67, 54, 41, 28, ...
4. 5.7, 6.9, 8.1, 9.3, ...
5. 4/9, 1/6, 1/16, 3/128, ...
6. 11a + 7b, 7a + 2b, 3a - 3b, -a - 8b, ...
7. 2, -5, 8, -11, ...
8. Suppose you are a frog with the following experience: A frog wishes to get to the top of a 49 ft. muddy slope inclined at 52°. He jumps three feet only to slide back one foot each time he lands on the incline. He traverses the slope in 24 jumps. Counting jumps and slides,



   a) express the total distance S_{24} as the sum of an arithmetic progression using the necessary problem data, and

   b) write the most likely name of the frog.

9. An ill-prepared but capable and industrious student made 35 on the first of seven algebra tests. What was his average grade if he made 10 more on each quiz than he did on the immediate preceding one?
10. Show that ka, kb, kc is an arithmetic progression (AP) if a, b, c is an AP.

Check your progress by comparing your answers with the answers given in the Self Evaluation Appendix. Never mind 8b for your 100%.

☐ OBJECTIVE 16.3

Can select geometric sequences from a list of sequences, determine their common ratio and calculate the sum of the first n terms.

Activities 16.3 (We suggest #1 and #2)

Note again the action verbs and that to determine or calculate depends on being able to identify those sequences which are geometric.

☐ 1. Your Text:
Reading Set 14.3, pages 322-324
Exercise Set 14.3, problems 21-40, page 325

☐ 2. Solved Problems:
Schaum's Outline: College Algebra. Reading Set, pages 140-141. Exercise Set: problems 25-44, pages 145-148.

☐ 3. Other Reading Sources:
Same as 16.1.

☐ 4. Lecture.

☐ 5. Informal Group Work.

☐ 6. Individual Assistance.

☐ Self Evaluation 16.3

Competency: 100%

Determine which of the following sequences are geometric progressions and find the common ratio r, S_5 and the 20th term for each such progression for problems 1-7.

1. 2, 6, 18, 54, ...
2. 1, 1/5, 1/25, 1/125, ...
3. $x - y$, x, $x + y$, $x + 2y$, ...
4. 64, -32, 16, -8, ...
5. 1, 1.03, $(1.03)^2$, $(1.03)^3$

6. 2, 4, 8, 12, ...

7. $1, 1 + a, 1 + 2a + a^2, 1 + 3a + 3a^2 + a^3, \ldots$

8. For what values of k are $2k$, $5k + 2$, and $20k - 4$ consecutive terms of a geometric progression?

9. The number of bacteria in a culture doubles every two hours. If there were n bacteria present at 5 a.m. one day, how many were there at 5 a.m. the next day?

10. Show that the quotients of corresponding terms of two geometric progressions forms a geometric progression.

☐ OBJECTIVE 16.4

Can determine specific terms and the general nth term of sequences.

Activities 16.4 (We suggest #4)

Actors are born, not made. Your intuition will play a leading role in this production, making any instructional package either easy or difficult to follow with no "in-between." No instruction could be found to refer you to at the writing of this manual so a programme with audio tape has been designed. If you should find an instructional procedure you feel is successful, inform your instructors so that it can be shared.

☐ 1. Your Text:

You can use your text with some effectiveness if you will consider the following scheme. Exercise 14.2, pages 320-321 is addressed to proof by induction which is covered in the next objective. If you will copy on another paper the left member except the general term of problems 1-10, you can use these problems for practice by writing the general term. For example: problem 1. $\underline{1 + 2 + 3 + 4 + \cdots} + n = \frac{1}{2} n(n + 1)$. On your paper write the terms 1, 2, 3, 4 and after a few moments, so you aren't cued on the answer, write the general term. (in this case "n")

☐ 2. Solved Problems:

Schaum's Outline: College Algebra. Follow the scheme suggested in working problems in your text for problem 1 of page 142 and problem 25, page 145 or make up your own by choosing a general term, generating four or so terms from the general term, wait a few moments so your response is not cued and work with the four terms you generated to arrive at the correct general term.

☐ 3. Other Reading Sources:

Almost all college algebra texts have a chapter on sequences and series. Use the scheme on problems of the type as suggested in #1 for practice.

☐ 4. Cassette Tape:

16.4. This programme uses a set of sequences and discusses the thoughts used to discover the pattern in each of the sequences.

☐ 5. Lecture.

☐ 6. Informal Group Work.

☐ 7. Individual Assistance.

☐ Self Evaluation 16.4

Competency: 9 out of 10.

Directions: Write the fifth term and the general term for each of the following sequences.

1. {1/4, 3/8, 9/16, 27/32, ...}
2. {2, 9, 20, 35, ...}
3. {0, -3, 8, -15, ...}
4. {2, 7, 12, 17, ...}
5. {3/2, 2/3, 5/12, 6/20, ...}
6. {-2, 5/8, -10/27, 17/64, ...}

☐ OBJECTIVE 16.5

Can apply mathematical induction to prove that the sum of the first n term of a given sequence is equal to a given algebraic expression in n, for all positive integral n.

Activities 16.5 (We suggest #1 and #2)

☐ 1. Your Text:

Reading Set 14.2, pages 318-320

Exercise Set 14.2, pages 320-321

☐ 2. Solved Problems:

Schaum's Outline: College Algebra. Chapter 18, pages 163-166 and the problems of page 166. A very good treatment.

☐ 3. Other Reading Sources:

Vance: Chapter 11, pages 220-226.

Other texts: Use the index or table of contents, cueing on "mathematical induction," "induction" or "types of proofs."

☐ 4. Lecture.

☐ 5. Informal Group Sessions.

☐ 6. Individual Assistance.

☐ Self Evaluation 16.5

Competency: 100%

Use the axiom of finite induction to prove the following.

1. $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$
2. Prove that the integer $7^{2n} - 1$ is divisible by 48 for every positive integer n.
3. $3^n \geq 1 + 2n$

4. $a^n - b^n$ is divisible by $a - b$

   or

   De Moivre's theorem: $r(\cos\theta + i \sin\theta]^n = r^n(\cos n\theta + i \sin n\theta)$

Check your proofs with those in the Self Evaluation Appendix.

ASSESSMENT TASK

Review the self evaluation exercises for each of the objectives. The module assessment is an instrument to measure each objective. You will need pencil and paper and a good night's rest before you take the assessment.

WHAT NEXT

If you demonstrated competency on the module assessment then you have completed the objectives of this level, that is if you are going for the "A". If Module 16 was chosen as an elective for the "B" then you should consider going for the A if time permits.

The next course in the sequence in the IPI mode of instruction is Math. 1813, Analytic Geometry. If you prefer this mode, ask your advisor for the correct section number when you enroll as it is presented conventionally and IPI.

You may elect to enroll in Math. 2215, Analytic Geometry and Calculus. This course is presented in the conventional instructional mode.

APPENDIX A

NOTES AND SOLUTIONS FOR SELF EVALUATIONS

APPENDIX A

NOTES AND SOLUTIONS FOR SELF EVALUATIONS

MODULE 6

Self Evaluation 6.1

Part I

1. F
2. F, a constant function
3. R
4. F
5. F
6. N, this is a set but not a set of ordered pairs
7. F
8. F
9. F
10. R, some men are not the "only brother of" i.e.; a male in the domain may have more than one brother in the range.
11. R
12. F
13. R, translation of "at most" is $\leq$.

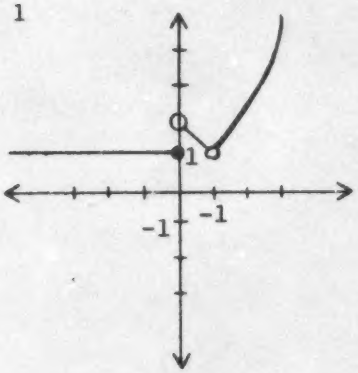
Part II

14. a) D(A) = {0, 3, 5, 1} in any order.
    b) D_e = {all reals} implied since not stated
    c) D_g or D(g) = {-1, 2, 3}
    d) D_j = {all reals except x = 1} or $D_j = \{x \neq 1\}$
15. a) R(B) = {1}
    b) R_g = {1, 7, 9}
    c) $R_h = 0 \leq y \leq 3$
    d) $I_{r(x)}$ = all reals greater than zero.

Self Evaluation 6.2

Part I

1



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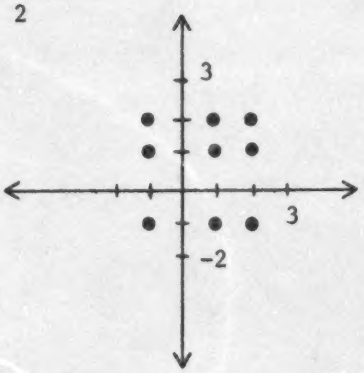

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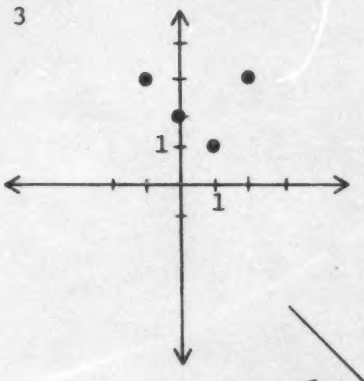

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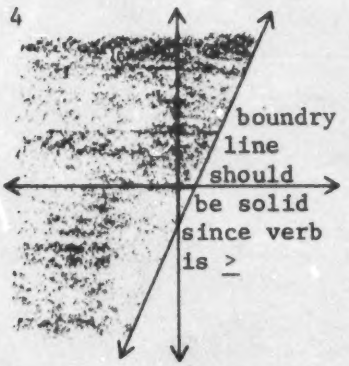
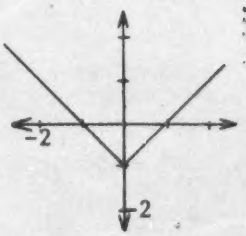




Part II

1, 2, 3, and 5 are not functions. #3 is questionable, however.

Self Evaluation 6.3

1. $f(x) + g(x) = 5x - 2, \quad -2 < x \leq 2.$ (Recall $D_{f+g} = D_f \cap D_g$)
2. $k(x) - h(x) = x^2 + 3x,$ {all reals}
3. $k(x)/i(x) = \frac{2 + 3x}{3/x}, \quad x \neq 0.$ (Or the simplified expression $1/3(2x + 3x^2), \quad x \neq 0.$)
4. $(h(x))(i(x)) = (2 - x^2)(2 + 3x)$ or $4 + 6x - 2x^2 - 3x^3,$ {all reals}

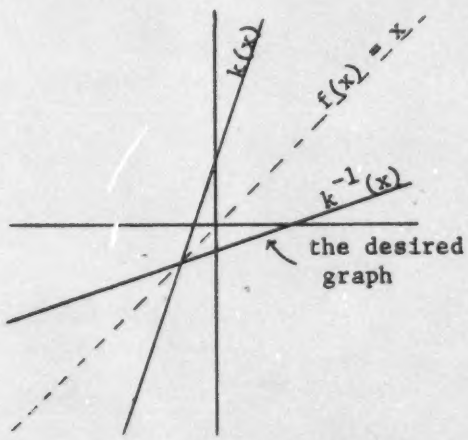
5. $(k^{-1}(x))(i(x)) + h(x) = \left(\frac{x-2}{3}\right)\left(\frac{3}{x}\right) + (2 - x^2)$, $x \neq 0$.
   Or $1/x(-x^3 + 3x - 2)$, $x \neq 0$.

6. $i(x)$: $y = \frac{3}{x}$ $\xrightarrow{\text{inverse}}$ $x = \frac{3}{y}$. $\therefore y = \frac{3}{x}$ is $i^{-1}(x)$.
   So $i^{-1}(x) = \frac{3}{x}$.

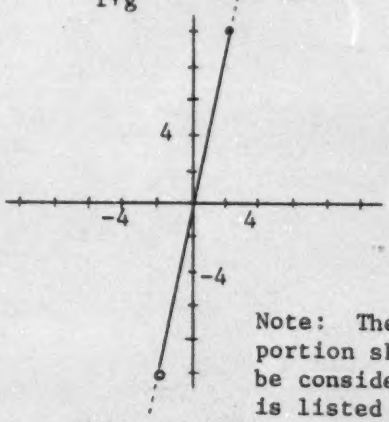
7. $h(x)$: $y = 2 - x^2$;
   $h^{-1}(x)$: $x = 2 - y^2$;
   solve for y yields $y = \sqrt{2 - x}$;
   therefore, $h^{-1}(x) = \sqrt{2 - x}$.

8. $T^{-1} = \{(3,1),(4,3),(4,2)\}$

9.



10. $(f + g)x = 5x - 2$
    $D_{f+g} = -2 < x \leq 2$



Note: The dotted portion should not be considered. It is listed only for a reminder that the graph in question is a subset of $f(x) = 5x - 2$.

Self Evaluation 6.4

1. $g(3) = 2(3) + 3 = 9$. So $f(g(3)) = f(g) = 4 - 9 = -5$.
   $f(g(3)) = -5$.

2. $f \circ g = f(2x + 3) = 4 - (2x + 3) = 1 - 2x$ with no restrictions on domain.

3. $g(h(x)) = 2(x^2 - 2x) + 3 = 2x^2 - 4x + 3$ with no restrictions.

4. If $f(r(x)) = g(x)$ or $for = g$, then $f^{-1} \circ for = f^{-1} \circ g$ if f^{-1} exists associative property holds. Therefore,

5. I_{hof}: $-1 < (hof)(x) < 3$. The range of f becomes the domain of h. A sketch of the graph of hof may help you see this.

$$(f^{-1} \circ f) \circ r = f^{-1} \circ g \quad \text{or} \quad r = f^{-1} \circ g$$

so to find $r(x)$ we find first f^{-1} and compose it with g.

$$f^{-1} = 4 - x.$$

Hence, $r(x) = 4 - (2x + 3)$ or

$$r(x) = 1 - 2x.$$

Since the range of r has no restriction, no restrictions are imposed on the domain of f so the composition has no restrictions.

MODULE 7

Self Evaluation 7.1

1. a) $2^5 = 32$ b) b^{3n} c) $1/\pi^5$

2. a) x^6 b) x^{11}/y^8

c) $$\frac{a^{-2} - b^{-2}}{a^{-1} - b^{-1}} = \frac{(a^{-1} + b^{-1})(a^{-1} - b^{-1})}{a^{-1} - b^{-1}} = a^{-1} + b^{-1}$$

[Note: $a^{-2} - b^{-2}$ is a "difference of squares" (although not in the usual form), and it may be factored as such.]

Self Evaluation 7.2

1. a) 1/4 b) 1/4 c) 9

2. a) $\frac{2}{3xy^2}$ b) $-\pi^{1/3}$ c) $\frac{\sqrt{a} + \sqrt{b}}{a - b}$ or $\frac{a^{1/2} + b^{1/2}}{a - b}$

NOTE: multiply by $\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}}$.

Self Evaluation 7.3

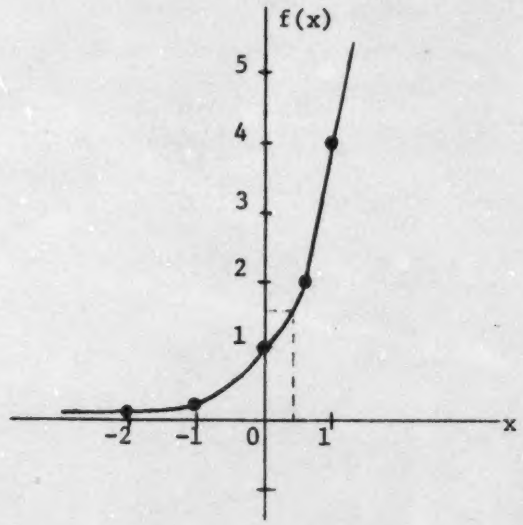
1. $x = -1/2$ 2. ϕ = the empty set 3. $x = -2$

4. $x = 4$ 5. $x = 1$, $x = 2$

Self Evaluation 7.4

1. a)

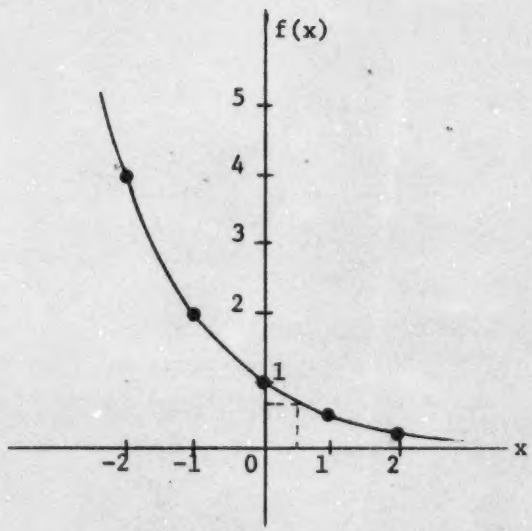
x	$f(x) = 4^x$
-2	1/16
-1	1/4
0	1
1/2	2
1	4



b) $4^{1/3} \approx 1.7$ (Note $4^{1/3} = f(1/3)$)

2. a)

x	$f(x) = (1/2)^x$
-2	4
-1	2
0	1
1	1/2
2	1/4



b) $(1/2)^{1/2} \approx 2/3 \approx 0.7$

Self Evaluation 7.5

1. a) $\log_a C = b, \quad a > 0, \quad a \neq 1$ b) $\log_2 32 = 5$

c) $\log_3 1/27 = -3$

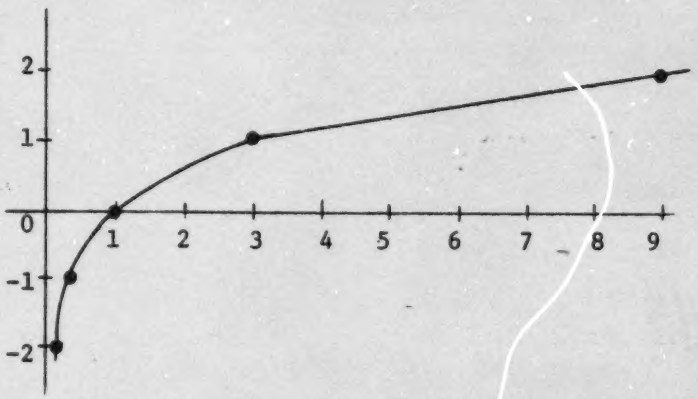
2. a) $\pi^b = a$ b) $(1/4)^2 = 1/16$ c) $4^{1/2} = 2$

3. a) $x = 2/3$ b) $x = 3$ c) $x = 1/32$

Self Evaluation 7.6

1.

x	$f(x) = \log_3 x$
1/9	-2
1/3	-1
1	0
3	1
9	2



NOTE: The values of x used in plotting points are integer powers of 3.

Self Evaluation 7.7

1. a) $\log_b x + \log_b y - \log_b z$

b) $3/2 \log_b x - 3/2 \log_b y - 3 \log_b z$

c) $2/3 \log_b x + 1/3 \log_b y - 1/3 \log_b z$

2. a) 1.0000 b) 0.0485

c) $\log_b 15 - \log_b b = 1.1761 - 1 = 0.1761$

3. a) $\dfrac{\log_2 25}{\log_2 4} = 1/2 \log_2 25$ b) $1/3 \log_2 7$

c) $-\log_2 2N$ or $-1 - \log_2 N$

Self Evaluation 7.8

1. a) 0.5105 b) 8.1818 - 10 c) 5.8470

2. a) 1.54 b) 8530 c) 0.07775 (approx)

Self Evaluation 7.9

1. 2.204 (approx)
2. 0.01325
3. 0.00362
4. 80.4 cubic feet

Self Evaluation 7.10

1. $x = 2.978$
2. $x = 3$
3. $x = 5$
4. $x = 1001$

MODULE 8

Self Evaluation 8.1

1. (1,0)
2. (-1,0)
3. (0,-1)
4. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Self Evaluation 8.2

1.

Function	Domain	Range
sin	all real numbers	$\{y \mid -1 \leq y \leq 1\}$
cos	all real numbers	$\{y \mid -1 \leq y \leq 1\}$
tan	$\{x \mid x \neq \frac{2k+1}{2}\pi,\ k = 0,\pm1,\pm2,\ldots\}$	all real numbers
cot	$\{x \mid x \neq k\pi,\ k = 0,\pm1,\pm2,\ldots\}$	all real numbers
sec	$\{x \mid x \neq \frac{2k+1}{2}\pi,\ k = 0,\pm1,\pm2,\ldots\}$	$\{y \mid y \geq 1\} \cup \{y \mid y \leq -1\}$
csc	$\{x \mid x \neq k\pi,\ k = 0,\pm1,\pm2,\ldots\}$	$\{y \mid y \geq 1\} \cup \{y \mid y \leq -1\}$

2. a) 1 b) $-\frac{1}{2}$ c) $-\sqrt{3}$

d) $\sqrt{2}$ e) -1 f) $-\frac{2\sqrt{3}}{3}$

3. a) 0.7660 b) 0.0204 c) -1.116

4. a) 0.9636 b) -0.9428 c) 0.6552

Self Evaluation 8.3

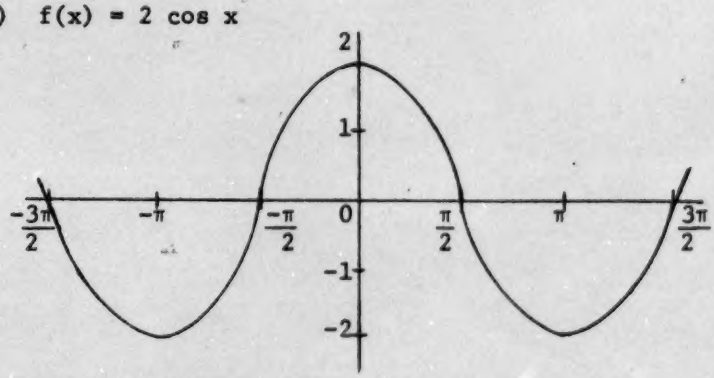
1. a) amplitude = 3, period = π

b) amplitude = 2, period = 4π

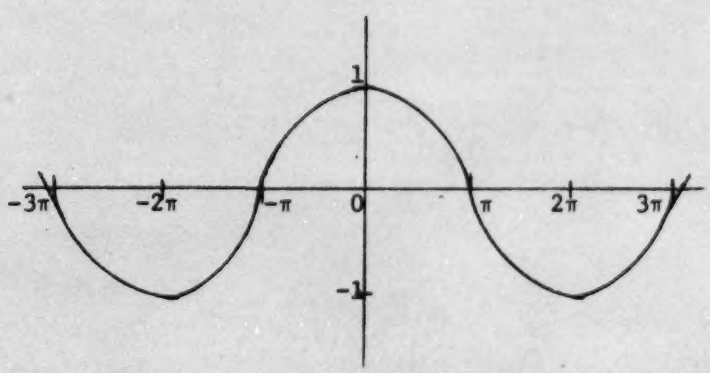
c) amplitude = ∞, period = $1/2\pi$

Self Evaluation 8.4

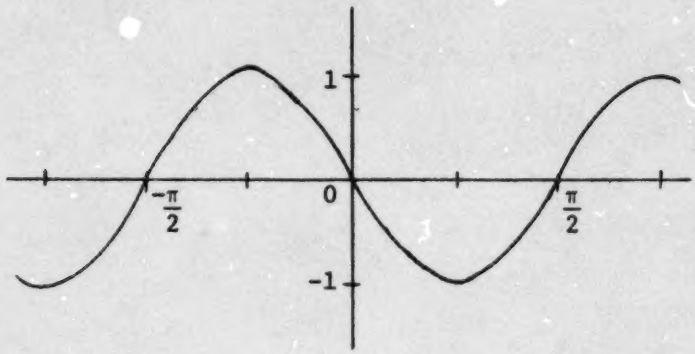
1. a) $f(x) = 2 \cos x$



b) $f(x) = \cos 1/2\ x$



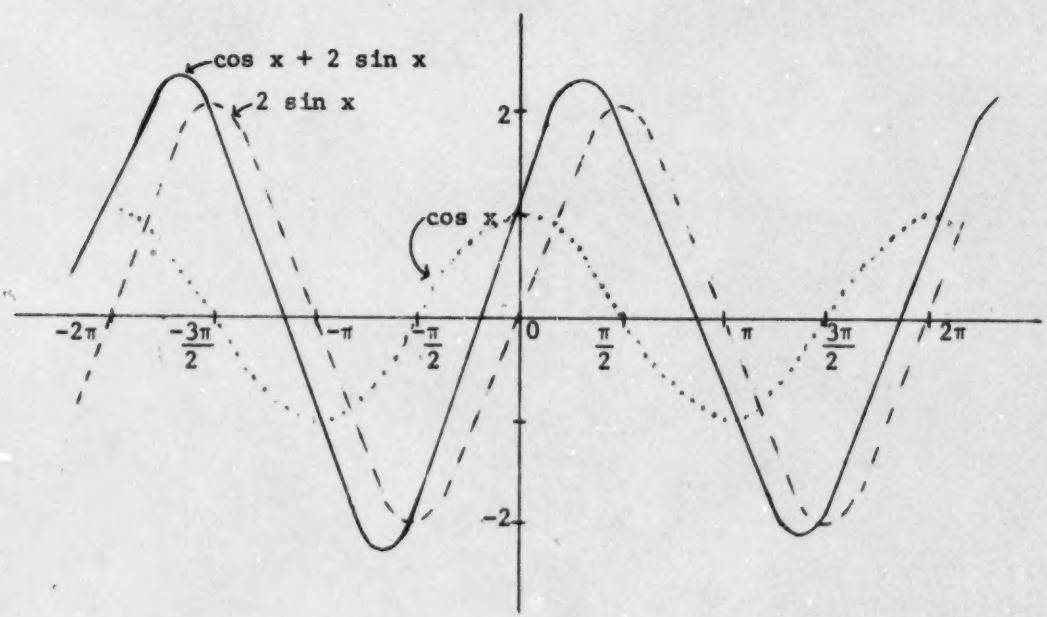
c) $f(x) = \cos(2x + \pi/2)$



2. a) amplitude = 3, period = 8π

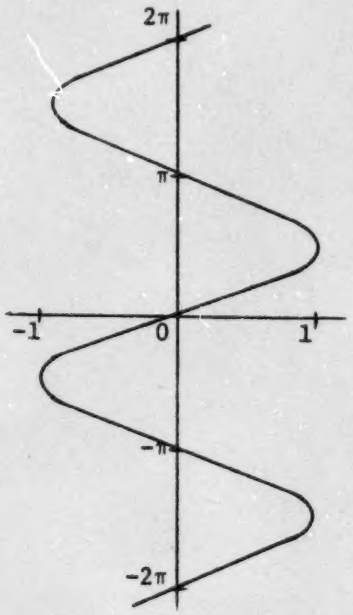
b) 3

3.



Self Evaluation 8.5

1.



2. $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

or

$\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

or

$\frac{k\pi}{2} \leq x \leq \frac{(k+1)\pi}{2}$,

$k = 0$ or ± 1 or ± 2 or ...

[NOTE: Any one of these restrictions may be utilized; the first restriction is the one most commonly used.]

MODULE 9

Self Evaluation 9.1

1. $-x^4 + x^3 - 6x^2 + 2x + x\sqrt{3} - 2$

2. $(f + g)(x) = x^5 + 5x^3 - 6x^2 + 3x - 6$

Self Evaluation 9.2

1. $6x^5 - 3\sqrt{5}x^4 + (3 - 2\sqrt{5})x^3 + 5x^2 - x\sqrt{5}$

2a. $ab\,x^4 + (a^2 + a^2b + b^2)x^3 + (a^2 + a^3 + ab + ab^2)x^2 +$
$+ (a^3 + a^2b + ab)x + a^2b$

2b. $x^3 + 3x^2 - 8x + 10$

Self Evaluation 9.3

1. a)
$$\begin{array}{rrrrr} & 1 & 0 & -5 & 2 \\ 1.5 & & 1.5 & 2.25 & +4.125 \\ \hline & 1 & 1.5 & -2.75 & +6.125 \end{array}$$
(Note the missing quadratic term)
$\therefore f(-1.5) = +2.125$

b)
$$\begin{array}{rrrrr} & 1 & 0 & -5 & 2 \\ -8 & & -8 & 64 & -472 \\ \hline & 1 & -8 & 59 & -470 \end{array}$$
$\therefore f(-8) = -470$

c)
$$\begin{array}{rrrrr} & 1 & 0 & -5 & 2 \\ 0 & & 0 & 0 & 0 \\ \hline & 1 & 0 & -5 & 2 \end{array}$$
$\therefore f(0) = 2$

There is an easier way.

2.
$$\begin{array}{rrrrrr} & 1 & -5 & 8 & -12 & 8 \\ +2 & & 2 & -6 & 4 & -16 \\ \hline & 1 & -3 & 2 & -8 & -8 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$
$$f(x) = x^4 - 5x^3 + 8x^2 - 12x + 8 = x^3 - 3x^2 + 2x - 8 + \frac{-8}{x - 2}$$

3. It had better be "zero" huh? By the factor or remainder theorem, which?

Self Evaluation 9.4

1. a) $\left\{ -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 2 \right\}$

b) trying the maximum and minimum possible rational zeros (± 2) happens to yield an upper and lower bound (not to be confused with least upper bound and greatest lower bound)

c) $\left\{ -\frac{1}{2}, \frac{1}{2}, 1 - i, 1 + i \right\}$ after you found $\pm \frac{1}{2}$ the depressed function is quadratic and is simple to find the other two complex zeros.

2. One method: $2 - i$ is a zero. Therefore, $2 + i$ is also a zero. Hence, $x^2 - 4x + 5$ is a factor of the polynomial. -3 is a root of multiplicity 2 then $(x + 3)(x + 3)$ or $x^2 + 6x + 9$ is a factor. Multiplying

$$(x^2 - 4x + 5)(x^2 + 6x + 9) = x^4 + 2x^3 - 10x^2 - 6x + 45$$

the desired polynomial.

3. $x^2 - 2ax + (a^2 + b^2)$

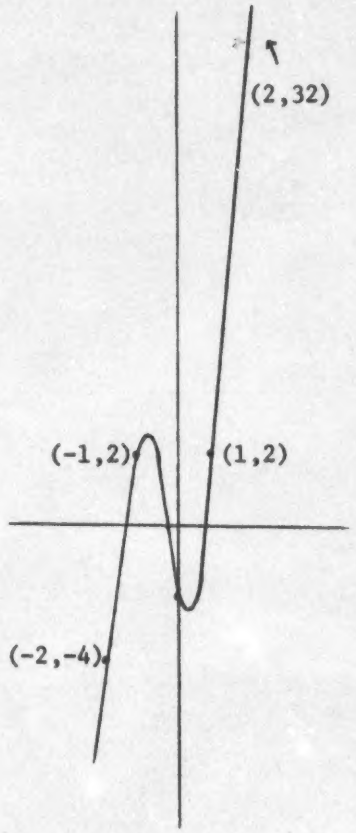
4. $\{a + bi, a - bi\}$

Self Evaluation 9.5

1.

r	3	4	-3	-2	(r,f(r))
-2	3	-2	1	-4	(-2,-4)
-1	3	1	-4	2	(-1,2)
0				-2	(0,-2)
1	3	7	4	2	(1,2)
2	3	10	17	32	(2,32)

Note that the constant term -2 allows only integral zeros of ±2,±1,0. The alternation of signs when -2 is applied inform us of a lower bound and +1 informs us of an upper bound. We know, therefore, with this information and the degree of the polynomial we have the heart of the graph.



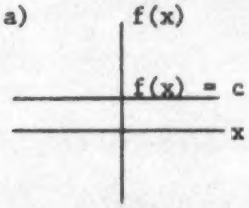
2. The only positive zero as indicated by the graph is between 0 and +1. By guestimation we try 0.7. f(0.7) = -1.111 indicating the zero now has to be between 0.7 and 1.0. Trying 0.9 yields 0.727. Continuing our hopping f(0.8) is 0.314 so the zero is between 0.7 and 0.8.

Applying the method of successive approximation

$$\frac{h}{0.1} = \frac{|-1.111|}{|-1.111| + |0.314|}, \quad h \approx .07.$$

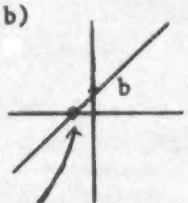
Hence, the positive zero is at .7 + .07 or 0.77 to nearest hundreth.

3. a)



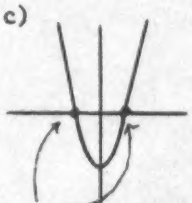
no zeros

b)

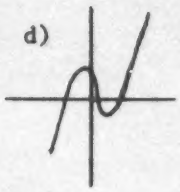


one real zero
f(x) = mx + b
m: slope
b: intercept

c)



two real zeros in this case. (Maybe none)
$f(x) = ax^2 + bx + c$

d) 

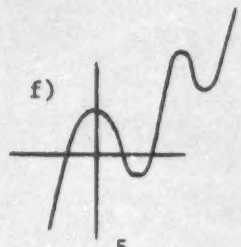
three real zeros in this case

$f(x) = ax^3 + a_1x^2 + a_2x + a_3$

e)

$f(x) = ax^4 + a_1x^3 + \cdots a_4$

two real zeros in this case. Could be 0, 2 or 4.

f) 

$f(x) = ax^5 + \cdots a_5$

3 real zeros here but could be 5. In any event there are an odd number of real zeros.

MODULE 10

Self Evaluation 10.1

1. $\frac{\sqrt{6} - \sqrt{2}}{4}$

2. $\frac{\sqrt{2}}{2 + \sqrt{2}} = \sqrt{2} - 1$

3. $\frac{119}{169}$

4. cos u

Self Evaluation 10.2

1. conditional equation: $x = \theta + 2k\pi$, $\theta = \pi/6$ or $5\pi/6$, $k = 0, \pm1, \pm2, \ldots$

2. conditional equation: $x = \theta + 2k\pi$, $\theta = \pi/2, 3\pi/2, 2\pi/3$, or $4\pi/3$, $k = 0, \pm1, \pm2, \ldots$

3. identity: $\frac{1 - \sin x}{\cos x} = \frac{\cos x(1 - \sin x)}{\cos^2 x} = \frac{\cos x(1 - \sin x)}{1 - \sin^2 x}$

$= \frac{\cos x(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} = \frac{\cos x}{1 + \sin x}$.

MODULE 11

Self Evaluation 11.1

1.

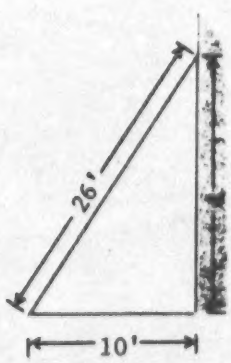


Figure 1

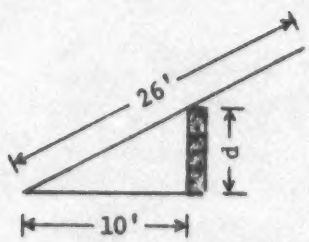
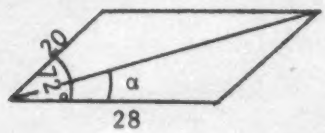


Figure 2

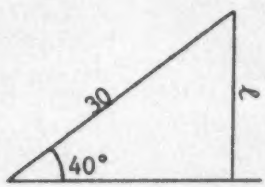
Note: The spirit of the problem implies Figure 1. On the basis of the data given, however, the diagram may appear similar to Figure 2; and the information is not sufficient to yield a numeric answer.

2.



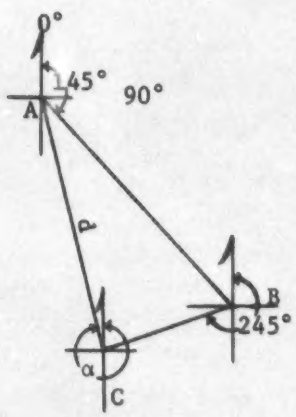
where α is the angle formed by the resultant and the side 28. (the angle needs to be specifically identified with one of the sides.)

3.



ℓ is the value sought and is measured in pounds.

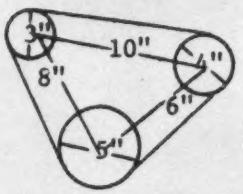
4.



d = CA, the distance

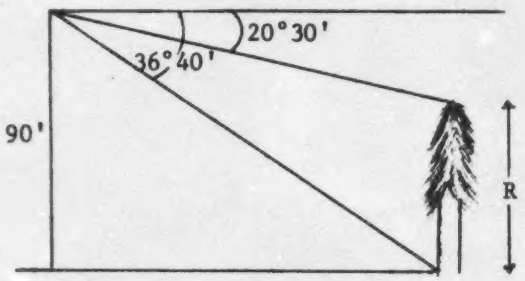
α is the heading requested.

5.



Here is a sketch, but it may make a difference if the pulleys were interchanged with respect to the vertices of the 6-8-10 triangle.

6.



Self Evaluation 11.2

Part I

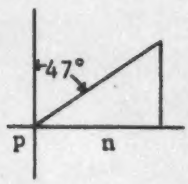
1. $\sin m = 3/4$ or $m = \sin^{-1} 3/4$
2. $m = \frac{5 \sin 48°}{\sin 104}$
3. $m = \sqrt{3^2 + 7^2 - 2(3)(7)\cos 65°}$
4. $m = \sqrt{4^2 - 5^2 + 2(5)(4)\cos 40°}$ or $\frac{4}{\sin 40°} = \frac{5}{\sin k}$ where k is the measure of the angle opposite the side of 5. One two or no choices for k will exist since this is an example of the ambiguous case.
5. $m = \frac{8 \sin 75°}{43}$ or apply the law of cosines
6. $m = \frac{1}{2}(7)(5)\sin 22°$
7. $\tan 70° = \frac{m}{4}$ or $m = 4 \tan 70°$

8. $\cos m = \dfrac{2^2 - 4^2 - 3^2}{-2(4)(3)}$

9. $45 = \frac{1}{2} n \left(\dfrac{n \sin 80°}{\sin 40°} \right) \sin 60°$

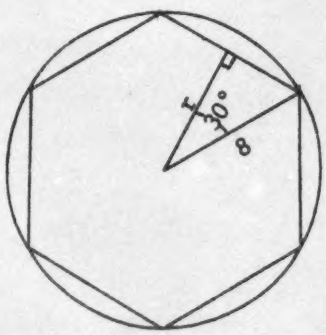
Part II

1.



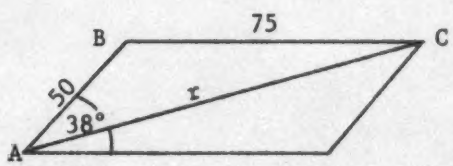
$\sin 43° = \frac{m}{45}$ is one of many possible setups. All the information given is not needed.

2.



$r = \dfrac{8}{\cos 30°}$ or $r = 8 \sec 30°$

3.



Use ΔABC and apply the law of cosines.
$r^2 = 50^2 + 75^2 - 2(50)(75)\cos 142°$

Self Evaluation 11.3

The solutions are written with the problems.

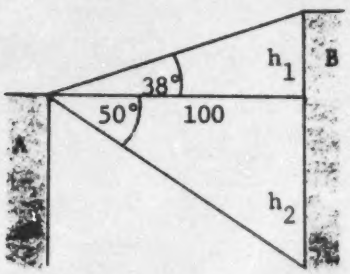
Self Evaluation 11.4

1. One method: $\tan \alpha = \frac{6}{5} \Rightarrow \alpha \approx 50°10'$

$\sin 50°10' = \frac{k}{15} \Rightarrow k \approx 15 \sin 50°10'$

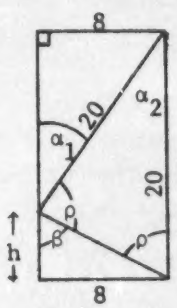
$k \approx 11.5$

2. $h_1 + h_2 = 100 \tan 38° + 100 \tan 50° \approx 197$ ft.



This is one of the simpler solution devices.

3.



$\sin \alpha_1 = \frac{8}{20} \quad \therefore \alpha_1 \approx 23°30'$

$\alpha_1 = \alpha_2$

$\alpha_2 + \rho + \rho = 180 \Rightarrow \rho \approx 78°15'$

$\alpha_1 + \rho + \beta = 180 \Rightarrow \beta \approx 78°15'$

$\cot \beta = \frac{h}{8} \Rightarrow h = 8 \cot \beta$

$h \approx 8(.204)$

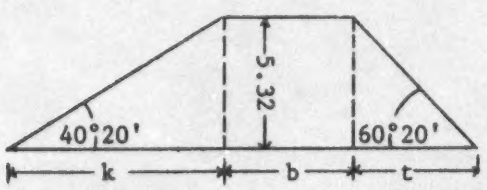
$h \approx 1.63$

4. One method: $A = \frac{1}{2} h(b + b')$

Applied: $A = \frac{1}{2} h(b + (b + k + t))$

$38.72 = \frac{1}{2} (5.32)(2b + 5.32 \cot 40°20' + 5.32 \cot 60°20')$

$b = 2.62$



MODULE 12

Self Evaluation 12.1

1. 12

2. $9 \times 10 \times 10 \times 10 \times 10 = 90{,}000$

3. $\frac{n(n-1)!}{(n+1)!} = \frac{n!}{n!(n+1)} = \frac{1}{n+1}$

4. 420

Self Evaluation 12.2

1. $P(7,7) = 7! = 5040$

2. $2P(5,5) = 2 \cdot 5! = 240$

3. $P(26,2) \times P(10,3) = 26 \times 25 \times 10 \times 9 \times 8 = 468{,}000$

4. $5! = 120$ Note: One person can sit anywhere, then the others may be permuted.

Self Evaluation 12.3

1. $C(20,2) = 190$

2. $C(10,5) = 252$

3. $10C(9,4) = (10)(126) = 1260$ or $5C(10,5) = 1260$

4. There are $C(7,3)C(8,4)$ ways of selecting a panel, and $P(7,7)$ ways of arranging each panel. Thus, the answer is $C(7,3)C(8,4)P(7,7)$.

Self Evaluation 12.4

1. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

2. The x^2y^2 term is the $(2x^2)^1y^2$ term, so the term is $C(3,2)(2x^2)y^2 = 3(2x^2)y^2 = 6x^2y^2$

3. $(2x - 5)^4 = (2x)^4 - 4(2x)^3 5 + 6(2x)^2 5^2 - 4(2x)5^3 + 5^4$
$= 16x^4 - 160x^3 + 600x^2 - 1000x + 625$

4. Since $C(7,3) = C(7,4)$, the terms are $C(7,3)x^4(y^2)^3 = 35x^4y^6$ and $C(7,4)x^3(y^2)^4 = 35x^3y^8$.

MODULE 13

Self Evaluation 13.1

1. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

2. $\frac{1}{10}$

3. $\frac{6}{10}$

4. $\frac{4}{10}$

5. $\frac{{}_5C_2}{{}_{10}C_2} = \frac{2}{7}$ Note: The notation ${}_nC_r$ is equivalent to the notation $C(n,r)$ used by the author of your text.

Self Evaluation 13.2

1. $\frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

2. $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$

3. $1 - \frac{4}{13} = \frac{9}{13}$

4. $\frac{1}{2} + \frac{4}{52} - \frac{2}{52} = \frac{1}{13}$

Self Evaluation 13.3

1. $\frac{3}{51}$
2. $\frac{47}{51}$
3. $\frac{25}{51}$
4. $\frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

Self Evaluation 13.4

1. $\left(\frac{1}{6}\right)^4$
2. $6\left(\frac{1}{6}\right)^4$
3. $\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)$
4. $C(4,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) + C(4,4)\left(\frac{1}{6}\right)^4$
5. $\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)\left(\frac{4}{6}\right)C(4,4)$

MODULE 14

Self Evaluation 14.1

1. $\left(2,-\left(\frac{1+2\sqrt{3}}{2}\right),4+2\sqrt{3}\right)$
2. $5 - \left(1 + \frac{\sqrt{3}}{2} + 8\sqrt{3}\right) = 4 - \frac{17\sqrt{3}}{2}$
3. $(2,-\sqrt{3},8)$
4. $(0,1,0)\cdot[(1,-3,4) + (4,-2,8\sqrt{3})] = -5$

5. $\sqrt{(\sqrt{3})^2 + \left(\frac{3}{2}\right)^2} = \frac{1}{2}\sqrt{21}$

6. $\left(\sqrt{3}, \frac{3}{2}\right) = \sqrt{(\sqrt{3})^2 + \left(\frac{3}{2}\right)^2}\left(\frac{\sqrt{3}}{\frac{1}{2}\sqrt{21}}, \frac{\frac{3}{2}}{\frac{1}{2}\sqrt{21}}\right)$

$= \frac{\sqrt{21}}{2}\left(\frac{2}{\sqrt{7}}, \frac{3}{\sqrt{21}}\right)$

Self Evaluation 14.2

1. $\begin{pmatrix} 3 & -3 \\ 2 & -2 \end{pmatrix}$

2. $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 3/2 & -3/2 \\ -5/3 & 4/3 & -4/3 \end{pmatrix}$

3. $\underset{CD}{\begin{pmatrix} 5/3 & 1/2 \\ 4/3 & -1/2 \end{pmatrix}} + \underset{B}{\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}} = \begin{pmatrix} 14/3 & -9/2 \\ 1/3 & 3/2 \end{pmatrix}$

4. $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

5. $\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & -6 \\ -9 & 22 \end{pmatrix}$

6. $g(A) = \underset{2A^2}{2\begin{pmatrix} 7 & -6 \\ -9 & 22 \end{pmatrix}} + \underset{3A}{3\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}} - \underset{-10I}{10\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Remark: Since $g(A) = 0$, then A is a zero of the polynomial.

7. b) DE and c) D^2 cannot be performed.

Self Evaluation 14.3

1. $x - y + 2w = 2$
   $3x - y + z - 2w = 1$
   $y + 2z + 2w = -3$
   $x - w = -1$

2. $\begin{pmatrix} 3 & -5 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

3. Since the coefficient matrix of S_1 is $\begin{pmatrix} 3 & -5 \\ 2 & 3 \end{pmatrix}$, a method to find the inverse employs the following scheme (without explanation or proof): $(A \mid I)$ transformed to $(I \mid A^{-1})$ where elementary row operations are used in the transformation.

$$\begin{pmatrix} 3 & -5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{pmatrix} \xrightarrow[\text{insert in } R_2]{-R_1 + R_2^{*}} \begin{pmatrix} 1 & -8 & 1 & -1 \\ 2 & 3 & 0 & 1 \end{pmatrix} \xrightarrow[\text{insert in } R_2]{-2R_1 + R_2} \begin{pmatrix} 1 & -8 & 1 & -1 \\ 0 & 19 & -2 & 3 \end{pmatrix}$$

$$\xrightarrow[\text{insert in } R_2]{\frac{1}{19} R_2} \begin{pmatrix} 1 & -8 & 1 & -1 \\ 0 & 1 & \frac{-2}{19} & \frac{3}{19} \end{pmatrix} \xrightarrow[\text{insert in } R_1]{8R_2 + R_1} \begin{pmatrix} 1 & 0 & \frac{-16 + 19}{19} & \frac{24 - 19}{19} \\ 0 & 1 & \frac{-2}{19} & \frac{3}{19} \end{pmatrix}$$

or $\begin{pmatrix} 1 & 0 & \frac{3}{19} & \frac{5}{19} \\ 0 & 1 & \frac{-2}{19} & \frac{3}{19} \end{pmatrix}$ So $A^{-1} = \begin{pmatrix} \frac{3}{19} & \frac{5}{19} \\ \frac{-2}{19} & \frac{3}{19} \end{pmatrix}$ or a more convenient form:

$A^{-1} = \frac{1}{19}\begin{pmatrix} 3 & 5 \\ -2 & 3 \end{pmatrix}$. $^{*}R_1$ means Row 1. $-3R_1 + R_2$ means multiply -3 times each entry in Row 1 and add to the corresponding entry in Row 2.

4. From Problem 2: $Ax = B$. Multiplying both members of this equation by A^{-1} (from Problem 3) and using the associative property of matrix multiplication

$$Ax = B \Rightarrow A^{-1}(Ax) = A^{-1}B \Rightarrow (A^{-1}A)x = A^{-1}B \Rightarrow Ix = A^{-1}B \Rightarrow x = A^{-1}B.$$

Thus $x = A^{-1}B$. Applied: $x = \frac{1}{19}\begin{pmatrix} 3 & 5 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix} \Rightarrow x = \frac{1}{19}\begin{pmatrix} 47 \\ 13 \end{pmatrix} \Rightarrow$

$x = \begin{pmatrix} \frac{47}{19} \\ \frac{13}{19} \end{pmatrix}$ So $x = \frac{47}{19}$, $y = \frac{13}{19}$

5. A similar procedure to Problem 4 $x = B^{-1}C$

$$x = \begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & 4 \end{pmatrix}\begin{pmatrix} 11 \\ 0 \\ 11 \end{pmatrix} \Rightarrow x = \begin{pmatrix} 88 + 0 - 33 \\ -55 + 0 + 22 \\ 110 + 0 + 44 \end{pmatrix} \quad x = \begin{pmatrix} 55 \\ -33 \\ 154 \end{pmatrix}$$

Apparently the inverse of B given is not correct since $x = 55$, $y = -33$, $z = 154$ does not satisfy the given system.

6. $\begin{pmatrix} 3 & -5 & 4 \\ 2 & 3 & 7 \end{pmatrix}$ the object of this method is to transform the coefficient matrix into I; then the entries in the constants column are the solution.

$$\left(\begin{array}{cc|c} 3 & -5 & 4 \\ 2 & 3 & 7 \end{array}\right) \xrightarrow[\text{insert in } R_1]{-R_2 + R_1} \left(\begin{array}{cc|c} 1 & -8 & -3 \\ 2 & 3 & 7 \end{array}\right) \xrightarrow[\text{insert in } R_2]{-2R_1 + R_2} \left(\begin{array}{cc|c} 1 & -8 & -3 \\ 0 & 19 & 13 \end{array}\right)$$

$$\xrightarrow[\text{insert in } R_2]{\frac{1}{19} R_2} \left(\begin{array}{cc|c} 1 & -8 & -3 \\ 0 & 1 & \frac{13}{19} \end{array}\right) \xrightarrow[\text{insert in } R_1]{+8R_2 + R_1} \left(\begin{array}{cc|c} 1 & 0 & \frac{104-57}{19} \\ 0 & 1 & \frac{13}{19} \end{array}\right) \text{ on}$$

$\left(\begin{array}{cc|c} 1 & 0 & \frac{47}{19} \\ 0 & 1 & \frac{13}{19} \end{array}\right)$ so $x = \frac{47}{19}$, $y = \frac{13}{19}$ which agrees with the results of Problem 4.

Self Evaluation 14.4

1. $\begin{vmatrix} 3 & -5 & -2 \\ 1 & 1 & 1 \\ 8 & 4 & 3 \end{vmatrix}$

2. Expansion by Row 1: $3\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} - (-5)\begin{vmatrix} 1 & 1 \\ 8 & 3 \end{vmatrix} + -2\begin{vmatrix} 1 & 1 \\ 8 & 4 \end{vmatrix}$

$3(-1) + 5(-5) - 2(-4) = -20$

3. $x = \dfrac{\begin{vmatrix} 20 & -5 & -2 \\ 3 & 1 & 1 \\ 27 & 4 & 3 \end{vmatrix}}{-20} = \dfrac{20\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} + 5\begin{vmatrix} 3 & 1 \\ 27 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ 27 & 4 \end{vmatrix}}{-20}$

$= \dfrac{20(-1) + 5(-18) + 2(-15)}{-20} = \dfrac{-80}{-20} = +4.$

Similarly, $y = -2$, $z = 1$.

Solving for y applying Cramer's rule given the value of the determinant of the coefficient matrix

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & 0 & 3 & 1 \\ 4 & 2 & 5 & -1 \\ 1 & 0 & -1 & -1 \end{vmatrix}}{36}$$

and expansion by column 2 since two zeros exist in column 2

$$\frac{(-1)^{1+0}\begin{vmatrix} 2 & 3 & 1 \\ 4 & 5 & -1 \\ 1 & -1 & -1 \end{vmatrix} + 0(-1)^{2+2}\begin{vmatrix} 1 & 1 & 2 \\ 4 & 5 & -1 \\ 1 & -1 & -1 \end{vmatrix} + 2(-1)^{3+2}\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix}}{36}$$

$$+ 0(-1)^{4+2}\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 4 & 5 & -1 \end{vmatrix}$$

$$36$$

$$= -\begin{vmatrix} 2 & 3 & 1 \\ 4 & 5 & -1 \\ 1 & -1 & -1 \end{vmatrix} + 0 - 2\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix} + 0$$

$$36$$

Expanding these 3rd order determinants in a fashion similar to problem 3 yields $-\frac{30}{36}$ or $y = -\frac{5}{6}$.

MODULE 15

Self Evaluation 15.1

1. $6 - 4\sqrt{-3}$

2. 2

3. (5,2)

4. $\frac{2 - 3\sqrt{5}}{9} - \frac{(6 + \sqrt{5})i}{9}$

5. $40 + 40i$

6. $\frac{34}{26} + \frac{38i}{26}$ or $\frac{17}{13} + \frac{19i}{13}$

7. $(x - 3iy)(i + 4) = 10 - 23i$

$(4x + 3y) + (x - 12y)i = 10 - 23i$

$\therefore 4x + 3y = 10$

$x - 12y = 23$ $\Rightarrow$ $x = \frac{645}{17}$, $y = \frac{81}{51}$

Self Evaluation 15.2

1.

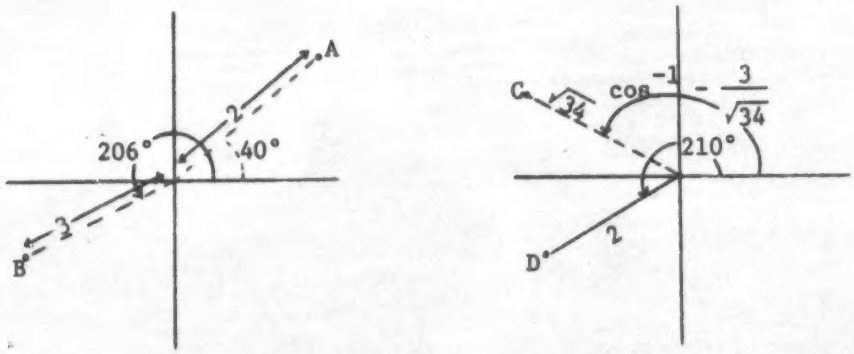
	Ordered Pair	Rectangular	Polar or Trigonometric	Misc.	Matrix
A	$(-\sqrt{3},-2)$	$-\sqrt{3} - 2i$	2 cos(-150 + i sin - 150) Change so angle is positive 2(cos 210° + i sin 210)		$\begin{pmatrix} -\sqrt{3} & 2 \\ -2 & -\sqrt{3} \end{pmatrix}$
B	(0,3)	3i	3(cos 90 + i sin 90)		$\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$
C	(2.12,-2.12)	.212 - 2.12i	3 cis 315°		$\begin{pmatrix} 2.12 & 2.12 \\ -2.12 & 2.12 \end{pmatrix}$

(Cont.)

D	$\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$	$\frac{5}{2} - \frac{5i\sqrt{3}}{2}$	$-5\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ Change so modulus is positive $+5\left(\cos\left(\frac{2\pi}{3}+\pi\right) + i\sin\left(\frac{2\pi}{3}+\pi\right)\right)$		$\begin{pmatrix} \frac{5}{2} & \frac{-5\sqrt{3}}{2} \\ \frac{5\sqrt{3}}{2} & \frac{5}{2} \end{pmatrix}$
E	$\left(\frac{2}{13}, \frac{3}{13}\right)$	$\frac{2}{13} + \frac{3}{13}i$	$\frac{\sqrt{13}}{3}\left[\cos\left(\tan^{-1}\frac{3}{2}\right) + i\sin\left(\tan^{-1}\frac{3}{2}\right)\right]$ or $.278(\cos 56°20' + i\sin 56°20')$	$\frac{1}{2-3i}$	$\begin{pmatrix} \frac{2}{13} & \frac{-3}{13} \\ \frac{3}{13} & \frac{2}{13} \end{pmatrix}$

2.

	Modulus	Amplitude
A	+2	40°
B	+3	206°
C	$+\sqrt{34}$	$\cos^{-1}\left(\frac{-3}{\sqrt{34}}\right)$
D	+2	210° or $\frac{7\pi}{6}$



3. $3[(\cos 30 + i\sin 30)][2(\cos 90 + i\sin 90)]$

$= \left[3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\right][2(0 + i)]$

$= 3\cdot 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)(0 + i)$

$= 6\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 6(1\cdot(\cos 120° + i\sin 120°))$

$= 6(\cos 120 + i\sin 120)$

4. Optional

Show $A \cdot B = B \cdot A$ if A & B are matrix representations are complex numbers. Note: in general, matrix multiplication is not commutative.

Let $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ and $B = \begin{pmatrix} c & -d \\ d & c \end{pmatrix}$

$$A \cdot B \Rightarrow \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c & -d \\ d & c \end{pmatrix} \xrightarrow[\text{multiplication}]{\text{by definition of matrix}} \begin{pmatrix} ac - bd & -ad - bc \\ bc + ad & -bd + ac \end{pmatrix}$$

Now cheat a little on scratch paper to find the appearance of $B \cdot A$

$\Downarrow$ commutative property of real numbers since the elements of the matrix are real

$$\begin{pmatrix} ca - db & -bc - ad \\ ad + bc & ac - bd \end{pmatrix}$$

$\Downarrow$

$$\begin{pmatrix} c & -d \\ d & c \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix} = B \cdot A$$

$\therefore A \cdot B = B \cdot A$ by the transitive of $\Rightarrow$

5. $2a(\cos\theta + i \sin\theta) = 5 + 2i$

One Method:

$$\text{then } 2a \sin\theta = 2 \\ \text{and } 2a \cos\theta = 5 \Rightarrow \frac{2a \sin\theta}{2a \cos\theta} = \frac{2}{5} \Rightarrow \tan\theta = \frac{2}{5} \Rightarrow \underline{\theta \approx 22^\circ}$$

$$\text{so } a \approx \frac{2}{2 \sin 22^\circ} \Rightarrow a \approx \frac{1}{\sin 22^\circ} \Rightarrow a \approx \csc 22^\circ \Rightarrow \underline{a \approx 2.67}$$

Self Evaluation 15.3

1. $-7 + 7i\sqrt{3}$
2. $\frac{-3\sqrt{2}}{2} - \frac{3i\sqrt{2}}{2}$
3. $-1 - i\sqrt{3}$

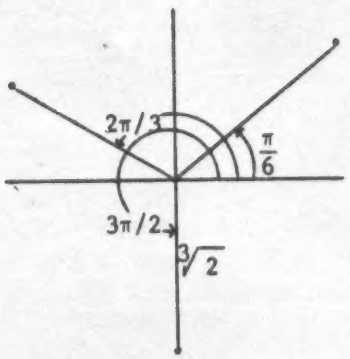
4. $\frac{1}{2} + \frac{i\sqrt{3}}{2}$

5. -1

6. $1, -1, i, -i$

7. $2, -i$

8.



The modulus in each case is the value of $\sqrt[3]{2} \approx 1.26$.

MODULE 16

Self Evaluation 16.1

1. a) $-1, -1, 0, 2, 5$

   b) $2, 4, 2, 4, 2$

2. a) $28 - 12b$ (10^{th})
   $43 - 32b$ (15^{th})

   b) 50 (10^{th})
   85 (15^{th})

3. $\begin{cases} a_1 = 4 \\ a_n = a_{n-1} + (n+1) \end{cases}$ or $\begin{cases} a_1 = 4 \\ a_{n+1} = a_n + (n+1) \end{cases}$

Self Evaluation 16.2

Problems 1, 3, 4, 6 describe an AP.

	d	S_{10}	a_{20}
1.	3	165	59
3.	-13	85	-66
4.	1.2	111.0	28.5
6.	-4a - 5b	-7a - 155b	-65a - 88b

8. $S_{24} = \frac{24}{2}[2(4) + 23(0)]$. Note: $a_n = \{4,4,4,\ldots\}$. Obviously $d = 0$.

9. Average $= \frac{\text{Total Points}}{\text{Number of Scores}} = \frac{S_7}{7} = \frac{\frac{7}{2}[2(35) + 6(10)]}{7} = 65.$

10. a,b,c form an AP $\Rightarrow$ $b - a = c - b$

$k(b - a) = k(c - b), \quad k \neq 0$

$kb - ka = kc - kb$

$\therefore$ ka,kb,kc form an AP.

Self Evaluation 16.3

	r	S_5	a_{20}
1.	3	242	$2(3^{19})$
2.	$\frac{1}{5}$	$\frac{781}{625}$	$\left(\frac{1}{5}\right)^{19}$

3. This is an AP with $d = y$.

4.	$-\frac{1}{2}$	44	$-\frac{1}{2^{13}}$
5.	1.03	1.1592740743	$(1.03)^{19}$

6. Not a GP. $r = 2$ for first 3 terms only.

7.	1 + a	$5 + 10a + 10a^2 + 5a^3 + a^4$	$(1 + a)^{19}$

8. $k = 2$ or $-\frac{2}{15}$

9. 4095n if none died and conditions ideal.

10. A rough sketch:

If a_n and b_n are GP, then $a_n = r(a_{n-1})$ and $b_n = r'(b_{n-1})$.

Then the sequence of quotients $\frac{a_n}{b_n} = \frac{r(a_{n-1})}{r'(b_{n-1})} = \frac{r}{r'}\left(\frac{a_{n-1}}{b_{n-1}}\right)$.

Since r, r' are constant, let $\frac{r}{r'} = R$. We have

$\frac{a_n}{b_n} = R\left(\frac{a_{n-1}}{b_{n-1}}\right)$ which indicates the nth term of sequence of quotients is a constant ratio times the preceeding term; and, therefore, a GP.

Self Evaluation 16.4

1. $\frac{1}{4}\left(\frac{3}{2}\right)^{n-1}$

2. $(n + 1)(2n - 1)$

3. $(-1)(n^2 - 1)$

4. $2 + (n - 1)5$

5. $\frac{n + 2}{n(n + 1)}$

6. $(-1)^n \frac{n^2 + 1}{n^3}$

Self Evaluation 16.5

1. P_1: $1 + 2^1 = 2^{1+1} - 1$

$3 = 3$ $\quad\therefore P_1$ true.

Assume P_k: $\quad 1 + 2 + 4 + \cdots + 2^k = 2^{k+1} - 1$

Add 2^{k+1} to both members:

$$\begin{aligned} 1 + 2 + 4 + \cdots + 2^k + 2^{k+1} &= 2^{k+1} + 2^{k+1} - 1 \\ &= 2(2^k + 2^k) - 1 \\ &= 2(2^k(1 + 1)) - 1 \\ &= 2(2^{k+1}) - 1 \\ &= 2^{k+1+1} - 1 \end{aligned}$$

Thus, P_{k+1}: $\quad 1 + 2 + 4 + \cdots + 2^{k+1} = 2^{(k+1)+1} - 1$

follows from P_k and P_n is true by induction.

2. P_1: If $n = 1$, then the integer is $7^2 - 1$ or 48.

P_2: $7^4 - 1 = 2400$ and is divisible by 48.

Therefore, 1 and 2 are in the set defined by $S = \{n \mid 7^{2n} - 1$ is divisible by $48\}$. Now show S is the set of all positive integers.

P_k: If $k \in S$, then $7^{2k} - 1$ is divisible by 48. Let q be the integer so that $7^{2k} - 1 = 48q$ or $7^{2k} = 1 + 48q$.

We need to show that 48 is a divisor of $7^{2(k+1)} - 1$.

$$\begin{aligned} 7^{2(k+1)} - 1 &= 7^{2k+2} - 1 \\ &= 7^{2k} \cdot 7^2 - 1 \\ &= (1 + 48q)7^2 - 1 \\ &= 7^2 + 7^2(48q) - 1 \\ &= 7^2 - 1 + 7^2(48q) \\ &= 48 + 7^2(48q) \\ &= 48(1 + 49q) \end{aligned}$$

Since $1 + 49q$ is an integer, the last equation shows 48 is a divisor of $7^{2(k+1)} - 1$. Therefore, $k+1$ is in the set S.

$$1 \in S$$
$$2 \in S$$

Whenever $k \in S$, $k+1$ also belongs to S. Therefore, $7^{2n} - 1$ is divisible by 48 for all positive integers n by induction.

3. P_1: $3^1 \geq 1 + 2(1)$ $\quad\therefore 1 \in S$ or P_1 true.

Assume P_k: $3^k \geq 1 + 2k$

$$3^k(3) \geq (1 + 2k)3 = 3 + 6k \geq 3 + 2k$$
$$= 1 + 2k + 2$$
$$= 1 + 2(k + 1)$$

Thus, P_{k+1}: $3^{k+1} \geq 1 + 2(k + 1)$ follows from P_k and P_n is true by induction.

4. a) $a^n - b^n$ divisible by $a - b$. P_1: $a^1 - b^1$ divisible by $a - b$. Assume P_k: $a^k - b^k$ is divisible by $a - b$. Add 0 expressed as $a^k b - a^k b$.

$$a^{k+1} - b^{k+1} = a^{k+1} - a^k b + a^k b - b^{k+1}$$
$$= a^k(a - b) + b(a^k - b^k).$$

Since the first term contains $(a - b)$ as a factor and is, therefore, divisible by b and the second term is divisible by $(a - b)$ from the assumption P_k, then $(a - b)$ divides the sum. Therefore,

$$P_{k+1}: \quad a^{k+1} - b^{k+1} = (a - b)[a^k + b(t)]$$

Since $a - b$ divides the right member, it must also be a divisor of the left.

$\therefore P_n$ is true.

b) P_1: $r(\cos\theta + i \sin\theta)]^1 = r'[\cos(1\cdot\theta) + i \sin(1\cdot\theta)]$ is true. Assume P_k true.

$$[r(\cos\theta + i \sin\theta)]^k = r^k(\cos k\theta + i \sin k\theta).$$

We wish to show

$$[r(\cos\theta + i \sin\theta)]^{k+1} = r^{k+1}(\cos(k+1)\theta + i \sin(k+1)\theta]$$

The (k+1)th power of a number

$$
\begin{aligned}
&[r(\cos\theta + i\sin\theta)]^{k+1} \\
&\quad = [r(\cos\theta + i\sin\theta)]^{k}[r(\cos\theta + i\sin\theta)] \\
&\quad = r^{k}(\cos k\theta + i\sin k\theta)[r(\cos\theta + i\sin\theta)] \\
&\quad = r^{k}\cdot r(\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta) \\
&\quad = r^{k+1}(\cos(k\theta + \theta) + i\sin(k\theta + \theta)) \\
&\quad = r^{k+1}[\cos(k+1)\theta + i\sin(k+1)\theta)
\end{aligned}
$$

$\therefore$ $k+1$ belongs whenever k belongs and the statement is true for all positive integer n by induction.

APPENDIX B

REFERENCES

APPENDIX B

REFERENCES

Text References

Eulenberg and Sunko: Introductory Algebra, 2nd Ed. John Wiley and Sons. New York, 1968.

This is the adopted text for Math. 1115 and Math. 1213, Intermediate Algebra. This text is available for sale in the bookstore. You can probably borrow this book from a friend.

Robison: Modern Algebra and Trigonometry. McGraw-Hill. New York, 1966.

This is your basic text. Reference to this book is made throughout the module activities. To be without a copy would make you as slow as if you were walking barefoot through a sandburr patch with a friend who had shoes. He would finally tire of carrying you and would be foolish to give you his shoes.

Vance: Modern College Algebra, 2nd Ed. Addison-Wesley. Reading, Mass., 1967.

This is the text adopted for use in College Algebra (Math. 1513) and is probably available to you through a friend.

Audio Tapes

Audio Tapes with programs for specific topics referred to in Study Guide. These are available in Physical Science Library.

Other Reading Sources

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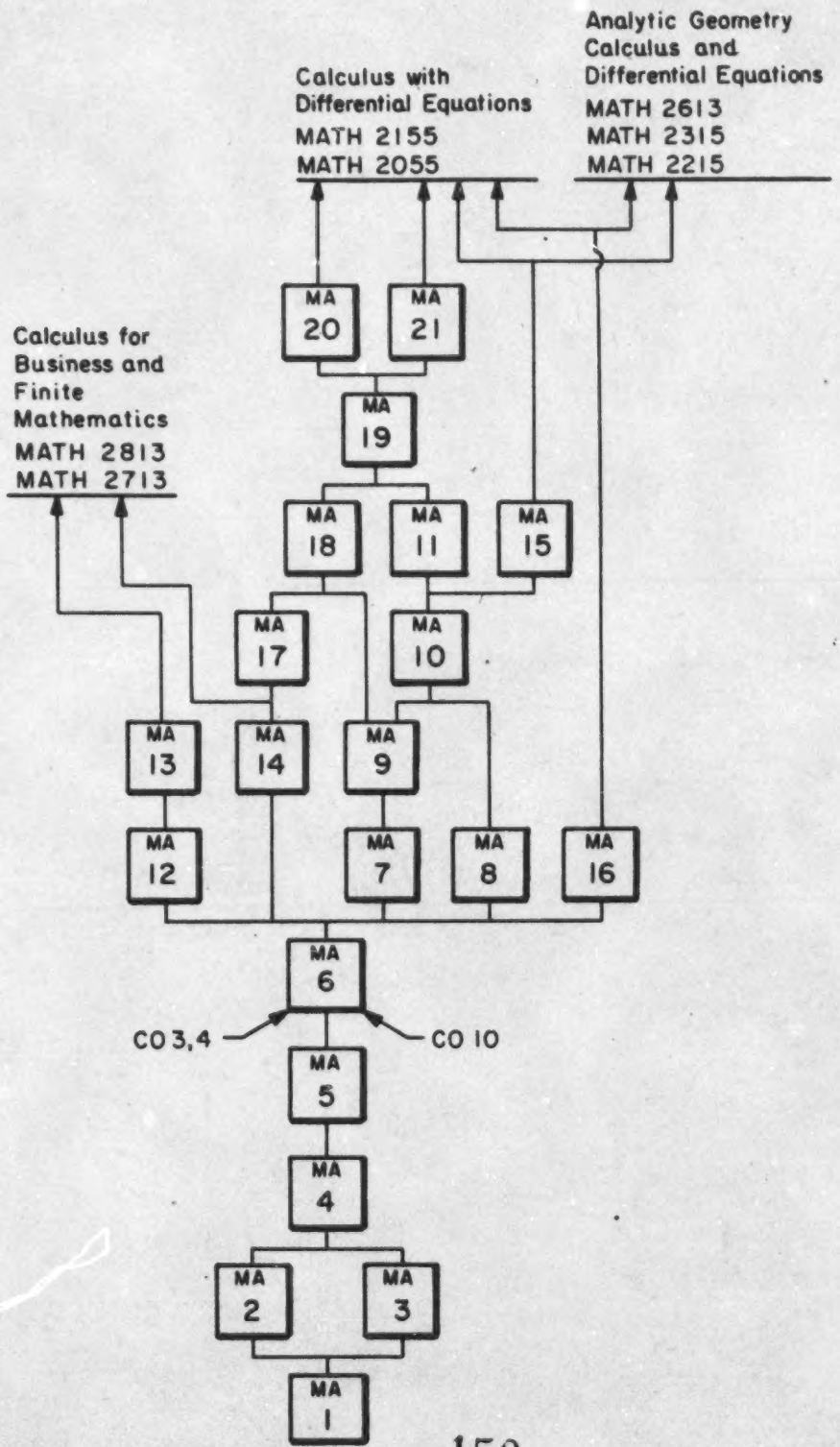
	PIPI Center	Library Open Reserve	Other

APPENDIX C

PRE CALCULUS MATHEMATICS

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